Teaching Quantum Uncertainty[1]

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An earlier paper[2] introduces quantum physics by means of four experiments: Young's double-slit interference experiment using (1) a light beam, (2) a low-intensity light beam with time-lapse photography, (3) an electron beam, and (4) a low-intensity electron beam with time-lapse photography. It's ironic that, although these experiments demonstrate most of the quantum fundamentals, conventional pedagogy stresses their difficult and paradoxical nature. These paradoxes (i.e., logical contradictions) vanish, and understanding becomes simpler, if one takes seriously the fact that quantum mechanics is the nonrelativistic limit of our most accurate physical theory, namely quantum field theory, and treats the Schroedinger wave function, as well as the electromagnetic field, as quantized fields.[2] Both the Schroedinger field, or "matter field," and the EM field are made of "quanta"—spatially extended but energetically discrete chunks or bundles of energy. Each quantum comes nonlocally from the entire space-filling field and interacts with macroscopic systems such as the viewing screen by collapsing into an atom instantaneously and randomly in accordance with the probability amplitude specified by the field. Thus, uncertainty and nonlocality are inherent in quantum physics. This paper is about quantum uncertainty. A planned later paper will take up quantum nonlocality.

The rationale behind this series[1] is that the conceptual difficulties of modern physics entail that physics teachers at all levels—high school, introductory college, and college undergraduate or graduate—communicate the fundamental concepts as clearly as possible, without unnecessary logical paradoxes.

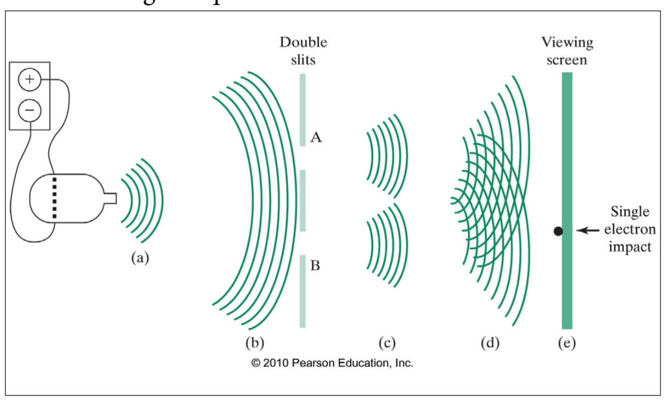
A quantum isn't a particle; it's a way of talking about energy increments of a spatially continuous EM field or matter field. Thinking again of the double-slit experiment, even if tiny individual impacts appear on a viewing screen, each quantum (each impact) is an energy increment of the entire space-filling field and hence comes, equally and symmetrically, through both slits. Because the field carries energy and momentum and collapses nonlocally to a small region of interaction (usually a single atom), each quantum hits like a particle. But it's not really a particle; it's a field that, after collapse, is spread out over a volume of approximately atomic dimensions. The particle-like increments of EM energy are called "photons," and the increments of matter field energy are called "electrons" (or protons, quarks, atoms, molecules, etc.).

For nonrelativistic electrons, the field equation for the matter field is Schroedinger's equation. Thus, the psi of Schroedinger's equation is not simply a mathematical probability amplitude; it's a physically real matter field, the nonrelativistic limit of the electron-positron field of quantum field theory. As Steven Weinberg puts it, "quantum fields are the basic ingredients of the universe, and the particles are just bundles of energy and momentum of the fields."[3] The universe seems (but isn't really) made of particles because the fundamental fields are quantized. Thus, in a wholly unexpected way, modern physics says that matter is both continuous and discrete. More precisely, it's made of discrete (countable) quanta of a continuous matter field.

In the rest of this paper I'll focus on matter rather than light, and on electrons because, being low in mass, they strongly demonstrate their quantum nature.

Quantum uncertainty

Figure 1 helps visualize an electron: A portion of an electron beam, containing only a single quantum (a single electron), is emitted at (a), approaches a double-slit experiment's slits and passes through them at (b) and (c) (reflected parts of the beam are not shown), approaches the screen in an interference pattern (d), and instantaneously collapses to an atom-sized region upon interacting with the screen at (e). Note that, before it hits the screen, each quantum (each electron) is highly de-localized, spreading across a broad region that includes both slits, and thus each electron responds to the entire experimental setup, including the presence of two open slits. This "wholistic" or "nonlocal" nature of quantum physics, a direct consequence of space-filling quantum fields, is behind much of the strangeness of quantum physics. The "choice" of which atom in the viewing screen to interact with is random, or uncertain, with a probability amplitude described by the psi of Schroedinger's equation.



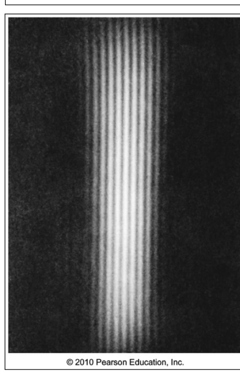


Fig. 1. Top, double-slit experiment with electrons, showing the matter field for a very low-intensity electron beam carrying only a single quantum of matter-field energy, at five different instants. At the instant of impact, the extended matter field vanishes and is replaced by a matter field of atomic dimensions.

Fig. 2. Left, the double-slit wave-interference pattern made by millions of electrons striking the viewing screen within a short time.[2]

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To prevent misconceptions, students need to understand that the electron *is* the spread-out wave (or field disturbance) shown in Fig. 1. It's a misconception to think that a tiny particle is somehow also present along with the wave. Furthermore, when the electron makes an impact on the screen in Fig. 1(e), it remains a spread-out wave, only now of much-reduced, atomic, dimensions.

Eventually a pattern such as Fig. 2 appears, formed by millions of individual electron impacts and filling a macroscopic portion of the screen.[2] Yet each impact came from an electron that was prepared identically to the other electrons. This is quite non-Newtonian. Newtonian physics teaches us that identical physical conditions lead to identical outcomes. When randomness arises in macroscopic situations such as flipping a coin, the different outcomes are known to be caused by different initial conditions such as differing thumb tensions. But contrary to Newtonian physics, there is an inherent uncertainty in microscopic events: identical conditions lead to different, and thus unpredictable, outcomes. Nature is inherently uncertain about what she will do next. Unlike Newtonian physics, the future is not encoded in the present.

Given the field nature of an electron, this randomness is suggested by symmetry. Here's why: Suppose, for simplicity, an electron beam passes through a single narrow slit. The beam spreads out after passing through the slit and impacts the viewing screen in a broad, fairly uniform band centered on the line from slit to screen. Suppose the beam is very dilute so that impacts appear on the screen one at a time. The matter field for each electron spreads out uniformly over a macroscopic region of the screen, enveloping an enormous number of atoms, yet when an interaction occurs it must occur at only one of these atoms. Since the field is uniform across a huge number of atoms, there is no apparent reason for an interaction to occur at one rather than another atom. Given this symmetry, it's not surprising that the precise location of the interaction within the beam region is random.

From our intuitively classical viewpoint, we are inclined to ask why nature is unpredictable. But from the standpoint of quantum physics, a better question is, "Why are macroscopic events generally predictable, at least in principle?" As we'll see, there's a straightforward answer to this question.

There's a difference between irreducible quantum randomness and the classical randomness of phenomena such as coin flips, where uncertainties arise from incomplete information that could in principle be completed to remove the uncertainties.

Although most quantum uncertainties are microscopic, they can be the trigger for easily observed macroscopic effects such as the click of a Geiger counter or the individual flashes that contribute to Fig. 2.

Students may have seen the tracks of electrons or other particles in high-energy physics experiments. The tracks look Newtonian because they are made by successive low-energy individual interactions between a high-energy (fast-moving)

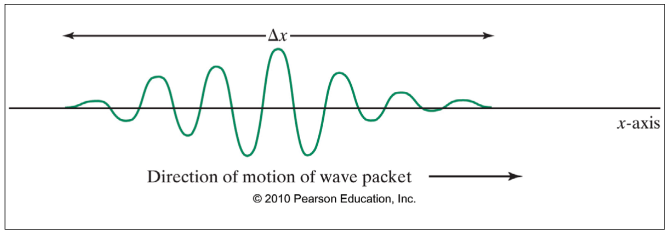

Fig. 3. A typical matter wave, or wave packet, for a single electron moving along the *x*-axis. Do not imagine that the electron is somewhere inside the wave packet. Rather, the wave packet *is* the electron. When it interacts with, say, an atom, the packet collapses to atomic dimensions.

field quantum and gas or water molecules. But uncertainty is still present: The quantum collapses each time it interacts with a molecule, while spreading out as a matter field between impacts.

Despite the randomness of individual quantum events, the statistics of large numbers of these events *are* predictable. For example, we get the same double-slit interference pattern every time we do the double-slit experiment. Overall statistics are predictable even though individual events are not. For nonrelativistic situations, Schroedinger's equation predicts these overall statistical patterns, just as Maxwell's equations predict the overall statistical patterns of large numbers of photons.

The uncertainty principle

Werner Heisenberg found, in 1927, that quantum uncertainties obey a certain quantitative rule. To get a feel for his argument, consider a quantized matter field containing enough energy for just one electron, moving through empty space in a direction that I will call the x-axis. Individual freely moving field quanta are generally spread out over some limited distance Δx (which is simply some measure of the spread of the quantum) and generally have a wave-like character. Figure 3 shows such a "wave packet." Remember that the wave packet is the electron. The wave packet is a single quantum of matter field energy, containing the electron's total energy and therefore its total inertial mass, as well as other features such as charge.

The Schroedinger equation predicts that a wave packet cannot sit still, and in fact cannot move with just a single velocity. It must instead move with a range of velocities. The reason is that a wave packet is a superposition of many different infinitely long waves, each having a definite wavelength (I'll let you decide whether to tell your students about Fourier analysis). But Louis de Broglie's formula, $\lambda = h/mv$, discovered in 1923 and confirmed by experiments with electron beams, says that different wavelengths correspond to different velocities. Thus, a wave packet has not only a range Δx of possible locations as shown in Fig. 3, but also a range Δv of possible velocities.

Let's compare one wave packet A with another packet B that has been squeezed into half of A's length, so that B's peaks and valleys are half as far apart as A's and Δx_B is half as large as Δx_A. B's shorter wavelengths mean that B represents a faster electron than A, in fact twice as fast because B's component wavelengths are half as big as A's and de Broglie's formula says v is inversely proportional to λ. This implies that Δv_B is also twice as large as Δv_A; for example, if A's velocity uncertainty ranges from 2000 to 3000 km/s, then B's velocity uncertainty must range from 4000 to 6000 km/s—twice as large as A's. In a conceptual physics course, you should replace these quantitative details with plausibility arguments.

This illustrates a general feature of quantum physics: Squeezing Δx (usually by making a measurement that determines the electron's position x to greater accuracy) tends to expand Δv, and vice versa, in such a way that the minimum value of the *product* of the two uncertainties remains unchanged. Heisenberg's detailed quantitative analysis showed that either uncertainty can take on any value, but the product of the two must be equal to or larger than $h/4\pi m$, where h is Planck's constant and m is the particle's mass. You can motivate the "or larger than" part by explaining that both Δx and Δv can always be arbitrarily large, simply for classical reasons (inaccurate measuring instruments), and that quantum physics places a limit only on how small the two can be.

There's no limit on how small Δx alone, or Δv alone, can be. Experiments show that a sufficiently accurate measurement can squeeze an electron's Δx to at least as small as 10^{-21} m, and physicists assume that electrons have zero size ("size" means "minimum Δx"). It's conceivable however that high-energy experiments, for instance at the Large Hadron Collider, could reveal a substructure of the electron, much as experiments at the Stanford Linear Accelerator revealed the quark substructure of protons and neutrons.

I'll refer to a particle's Δx and Δv as its "uncertainty range." You can visualize a particle's uncertainty range in a v-versus-x diagram (Fig. 4). Newtonian physics assumes that every particle has a precise x and v, and that larger objects such as a baseball can also be described with precise x's and v's; for instance, the center of a baseball has a precise x and v [Fig. 4(a)]. Newton's laws are essentially a method for predicting an object's future x's and v's from its present x's and v's.

But microscopic material particles are really quanta of a matter field, spread out chunks of field that do not have precise positions or velocities. In a v-versus-x diagram, a particle's uncertainty range appears as a rectangle whose area is $\Delta x \cdot \Delta v$ [Fig. 4(b)]. You can think of such a diagram as a rough portrait of a particle's matter field. Like other physical fields, a matter field is spread out over a range of positions and velocities. The uncertainty principle says that the area of this rectangle must be larger than $h/4\pi m$; you can squeeze either uncertainty as much as you like, but the other must always be sufficiently large to keep the total area larger than $h/4\pi m$

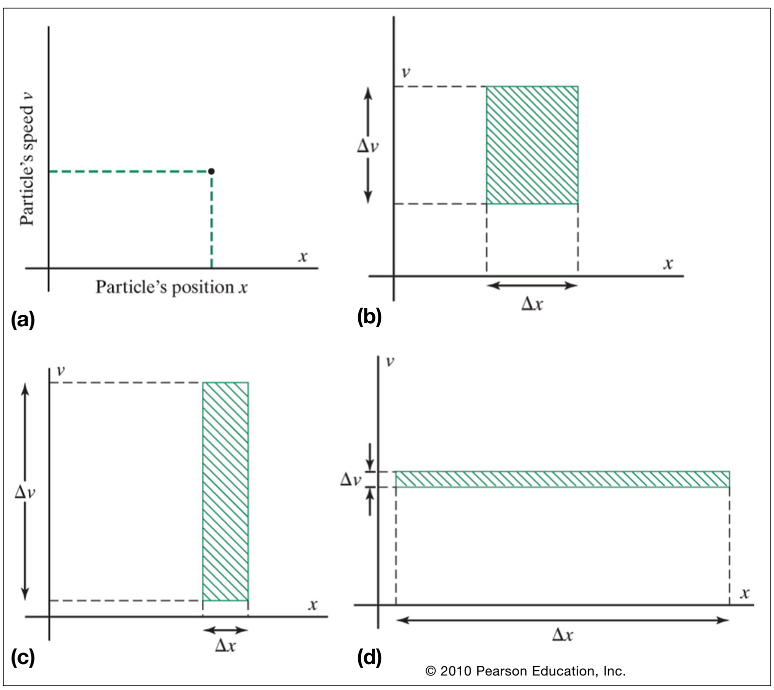

Fig. 4. Position and velocity uncertainty ranges. (a) A single point on an *x*-versus-*v* diagram represents a precise value of both *x* and *v*. Quantum theory does not allow such precise values. (b) An uncertainty range for a single particle. The shaded region must have an area $(\Delta x)\bullet(\Delta v) \geq h/4\pi m$. (c) If a measurement reduces Δx, and if this causes the area to drop below $h/4\pi m$, then Δv must increase to be large enough to maintain $(\Delta x)\bullet(\Delta v) \geq h/4\pi m$. (d) If Δv is reduced, and if this causes the area to drop below $h/4\pi m$, Δx must increase to be large enough to maintain $(\Delta x)\bullet(\Delta v) \geq h/4\pi m$.

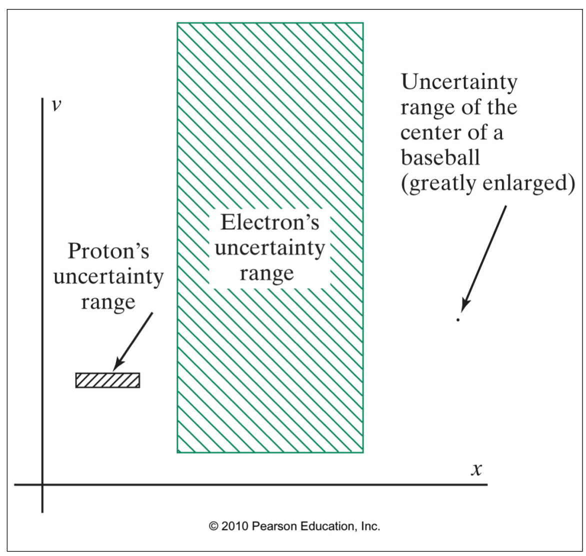

Fig. 5. More massive objects can have smaller uncertainty ranges.

[Figs. 4(c) and (d)]. When we (approximately) measure a particle's position, we partially collapse its Δx as in Fig. 4(c). Figures 1(d) and (e) show a common before-and-after example of this "collapse of the wave packet." Note that the measurement reduces, but doesn't remove, Δx. Even in Fig. 1(e), a nonzero Δx, of atomic dimensions, remains. Figure 4(d) represents the result of measuring a particle's velocity.

Since $\Delta x \cdot \Delta v \geq h/4\pi m$, more massive particles can have smaller uncertainty ranges. A proton, about 2000 times more massive than an electron, has a minimum uncertainty range 2000 times smaller in area than does an electron (Fig. 5). Because x and v are both needed to predict a particle's future behavior, a proton is more predictable than an electron. And a baseball, a million trillion trillion times more massive than an electron, is so predictable that quantum uncertainties are negligible (Fig. 5). That's one reason why the macroscopic world is Newtonian! Even a grain of sand contains 10^{18} atoms and is so massive that quantum uncertainties concerning such macroscopic properties as the location of its center of mass are negligible.

When a particle's Δx is squeezed into a sufficiently small range, it must develop a large Δv. But a large Δv implies a large speed; for instance, if Δv is 1000 km/s, then the slowest uncertainty range for v alone is -500 km/s to +500 km/s, and the average speed is at least 250 km/s, while if Δv is 2000 km/s, the average speed must be at least 500 m/s. So a highly confined particle (Δx small) must move fast. For example, the protons and neutrons in a nucleus must move on average at some 10% of light speed because nuclear forces confine them to such a tiny region. This is why the small-scale world is generally (but not always—unconfined atoms can, for instance, be made to move very slowly) a high-energy world.

Quantum uncertainties are of considerable practical importance. They might be exploited in future quantum computers. They lie at the heart of radioactive decay and cause this process to be fundamentally unpredictable. When a child is conceived, the DNA molecules of each parent are randomly combined in a process in which quantum physics plays a role. Thus, quantum uncertainty played a role in your genetic inheritance. And quantum uncertainties are written into the heavens: According to current "inflationary" theories of the big bang, the universe began from a quantum event and quickly expanded, freezing the quantum randomness that existed at early times into the distribution of matter at the largest scales and shaping the distribution of galaxies.

References

1. This article is one of a series about teaching modern physics that includes Art Hobson "Teaching $E=mc^2$: Mass without mass," *Phys. Teach.* **43**, 80–82 (Feb. 2005); Art Hobson "Teaching quantum physics without paradoxes," *Phys. Teach.* **45**, 96–99 (Feb. 2007); Art Hobson, "Teaching elementary particle physics: Part I," *Phys. Teach.* **49**, 12–15 (Jan. 2011); Art Hobson, "Teaching elementary particle physics: Part II," *Phys. Teach.* **49**, 136–138 (March 2011). Like the other articles in the series, this paper is based loosely on the author's liberal arts physics textbook *Physics: Concepts & Connections*, 5th ed. (Pearson/Addison-Wesley, San Francisco, 2010).
2. Art Hobson,"Teaching quantum physics without paradoxes," *Phys. Teach.* **45**, 96–99 (Feb. 2007); see also Art Hobson, "Electrons as field quanta: A better way to teach quantum physics in introductory general physics courses," *Am. J. Phys.* **73**, 630–634 (July 2005).
3. Steven Weinberg, in *Conceptual Foundations of Quantum Field Theory,* edited by Tian Yu Cao (Cambridge U.P., Cambridge, U.K., 1999), p. 242.

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Dr. Robert C. Hilborn to Serve as Associate Executive Officer of AAPT

Bob Hilborn

Following a national search, the American Association of Physics Teachers has announced the appointment of Dr. Robert (Bob) C. Hilborn as Associate Executive Officer.

Hilborn brings to the position extensive experience as a physics faculty member and college administrator at Oberlin College, Amherst College, the University of Nebraska-Lincoln, and the University of Texas at Dallas. With service in a variety of faculty and administrative positions, complemented by significant experience in leadership positions in undergraduate Science, Technology, Engineering, and Mathematics (STEM) education at the national level, he has a broad perspective and a track record of teaching, research, and administrative accomplishments.

Hilborn served as AAPT President in 1996-97 and worked with the Associate Executive Officer at that time. Following his tenure as AAPT President, he helped organize a workshop, topical conference, and physics department chairs meeting focusing on undergraduate physics at a time when the number of undergraduate physics degrees awarded each year was declining. In 1999, he co-organized the National Task Force on Undergraduate Physics to focus the physics community's attention on undergraduate physics and to assist physics departments that wish to enhance their undergraduate physics programs resulting in the SPIN-UP report, used by many physics departments as a guide for enhancing their undergraduate physics programs.

He is Principle Investigator on AAPT's Physics and Astronomy New Faculty Workshop grant from the NSF. This program is now serving about 50% of the new hires in physics and astronomy across the country. Hilborn also served on the writing team for *Active Physics* and helped develop the UTeach Dallas program to encourage STEM majors to go into K-12 teaching.

His experience in physics, physics education, K-12 STEM education, budgeting, management, proposal writing, and grant management make Hilborn a valuable addition to the AAPT staff. The Executive Board and staff welcome him and look forward to this new opportunity to work together.