Two-photon interferometry and quantum state collapse

Art Hobson*

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA

(Received 8 January 2013; revised manuscript received 13 March 2013; published 8 August 2013)

The quantum measurement problem still finds no consensus. Nonlocal interferometry provides an unprecedented experimental probe by entangling two photons in the "measurement state" (MS). The experiments show that each photon measures the other; the resulting entanglement decoheres both photons; decoherence collapses both photons to unpredictable but definite outcomes; and the two-photon MS continues evolving coherently. Thus, when a two-part system is in the MS, the outcomes actually observed at both subsystems are definite. Although standard quantum physics postulates definite outcomes, two-photon interferometry verifies them to be not only consistent with, but also actually a prediction of, the other principles. Nonlocality is the key to understanding this. As a consequence of nonlocality, the states we actually observe are the *local* states. These actually observed local states collapse, whereas the global MS, which can be "observed" only after the fact by collecting coincidence data from both subsystems, continues its unitary evolution. This conclusion implies a refined understanding of the eigenstate principle: Following a measurement, the actually observed *local* state instantly jumps into the observed eigenstate. We also discuss and rebut objections to this.

DOI: 10.1103/PhysRevA.88.022105 PACS number(s): 03.65.−w, 42.50.−p

I. INTRODUCTION

As Weinberg recently reminded us [1], the measurement problem remains a fundamental conundrum. To review the essentials, it is sufficient to consider two-state ("qubit") systems. Suppose a qubit S, whose Hilbert space is spanned by orthonormal states $|s_i\rangle$ ($i = 1,2$), is in the superposition state,

$$|\Psi\rangle_S = (|s_1\rangle + |s_2\rangle)/\sqrt{2}. \tag{1}$$

A measurement apparatus A, which may be micro- or macroscopic, is designed to distinguish between states $|s_i\rangle$ by transitioning into state $|a_i\rangle$ if it finds S is in $|s_i\rangle$ ($i = 1,2$) [2]. Assume the detector is reliable, implying the $|a_i\rangle$'s are orthonormal and that the measurement interaction does not disturb states $|s_i\rangle$—i.e., the measurement is "ideal" [3]. When A measures S, the Schrödinger equation's unitary time evolution then leads to the "measurement state" (MS),

$$|\Psi\rangle_{SA} = (|s_1\rangle|a_1\rangle + |s_2\rangle|a_2\rangle)/\sqrt{2}, \tag{2}$$

of the composite system SA following the measurement.

Standard quantum physics postulates a definite but unpredictable measurement outcome, either $|a_1\rangle$ or $|a_2\rangle$ and that S suddenly "collapses" into the corresponding state $|s_i\rangle$. Equation (2) does not appear to resemble such a collapsed state. The measurement problem is as follows: (1) How do we reconcile the measurement postulate's definite outcomes with the MS; and (2) how does the outcome become irreversibly recorded in light of the Schrödinger equation's unitary (and, hence, reversible) evolution? This paper deals with only the first part, known as the "problem of definite outcomes," a problem that Leggett [4] expresses as follows: "The quantum measurement paradox is that most interpretations of quantum mechanics at the microscopic level do not allow definite outcomes to be realized, whereas, at the level of our human consciousness it seems a matter of direct experience that such outcomes occur." The second part, which must take into account the macroscopic nature of either the apparatus or the wider environment, has been discussed in depth by Schlosshauer [2] and Zurek [5].

The MS is subtle: a bipartite (two-subsystem) entangled (nonfactorizable) coherent superposition. A crucial feature is that, like all entangled states, it implies a nonlocal connection between S and A [6]. This paper presents an experimental and theoretical analysis of the MS that resolves the problem of definite outcomes. It shows that the MS actually *is* the collapsed state of both S and A, even though it evolved purely unitarily. The formation of subsystems and the resulting "local" versus "global" distinction (a consequence of nonlocal entanglement) are crucial to understanding this: The *local* states of S and of A, i.e., the states that we actually observe directly at the locations of S and A (which could be light years apart), collapse into definite outcomes. The *global* state Eq. (2), which is observable only by the subsequent nonlocal collection of data (which could take years) showing the experimental coincidences between the two subsystems, maintains its unitary and reversible time evolution. Thus, local subsystem collapse is consistent with global unitary evolution of the composite system.

In 1990, two-photon interferometry experiments demonstrated a new way to experimentally test nonlocality (i.e., Bell's inequality) for entangled photon momentum eigenstates rather than, as in earlier nonlocality tests by Aspect and others, for entangled photon polarization eigenstates. These two-photon states were precisely the MS. Importantly, the phase angles φ_S between $|s_1\rangle$ and $|s_2\rangle$ and φ_A between $|a_1\rangle$ and $|a_2\rangle$ could both be varied independently at either subsystem. These experiments in which two photons (call them S and A) were entangled in the MS offer a unique window on the MS. The experiments were logically equivalent to two coupled double-slit experiments in which the entanglement caused each photon to act as a which-path (or momentum) detector for the other photon. We discuss these experiments' relevance to the measurement problem.

*ahobson@uark.edu

The experimental results (Sec. II) show that A decoheres S, whereas, S decoheres A, removing the interferences that would be seen if the photons were not entangled and collapsing both S and A as viewed locally into incoherent unpredictable but definite outcomes. This shows that the MS is the collapsed state exhibiting definite local outcomes. But at the global level, the experiment finds a two-body state with coherent nonlocal interferences. Thus, the experiments demonstrate the coherent unitary nature of the global evolution and the local collapse of S and A into definite outcomes.

Section III analyzes the MS theoretically. The MS's nonlocal aspect is crucial because experiment (Sec. II) and theory (Sec. III) both show that the states actually observed at both subsystems are the "reduced" states, not the global MS. These actually observed outcomes are not coherent superpositions but are, instead, incoherent and definite. The global MS remains a coherent superposition. Similar conclusions have been stated [7] and have been suggested [8,9], but these have been overlooked. Hopefully, the experimental facts and analysis presented here will reinforce these arguments. The "modal interpretations" [10–12] have long postulated these conclusions, but there is no need for a separate postulate because these results follow from the other standard quantum postulates.

Section IV rebuts previous analyses disputing these results, including misconceptions about Schrödinger's cat and the charge that decoherence cannot solve the measurement problem. Section V summarizes the conclusions.

II. THE MEASUREMENT STATE: EXPERIMENTS

In the ordinary two-slit experiment with a single photon, we see nonlocality in the photon's discontinuous and instantaneous jumps between a self-interfering pure state and a noninterfering mixed state depending, respectively, on whether a which-path detector is absent or is present. The instantaneous and nonlocal nature of these jumps is clear in the "delayed-choice" experiment [13]: A random decision to not entangle or to entangle the photon with a which-path detector is made quickly after the photon has entered the interferometer but before there has been time to send a signal to the detector. The photon expands to a superposition over both paths [14] the instant the detector switches off and collapses to a mixture the instant the detector switches on. This and similar recent experiments [15] leave little doubt, today, of instantaneous (or at any rate "faster than light") state collapse upon measurement.

Thus, the many nonlocality experiments during recent decades are likely to be relevant to understanding the MS. Nonlocal two-photon interferometry experiments by Rarity and Tapster [16,17] and, independently, Ou *et al.* [18] probe the MS. Such experiments, described below, can determine which states are incoherent with unpredictable but definite outcomes and which are coherent superpositions. Importantly, in these Rarity, Tapster, and Ou *et al.* (RTO) experiments, both S and A are microscopic, so both subsystems can be fully analyzed with rigorous quantum theory, and the phases of S and A can be varied. As we will see, the experiments demonstrate each local subsystem to be in an incoherent mixture of unpredictable but definite outcomes. But coincidences of paired photons demonstrate that the global state remains coherent. Thus, both subsystems are in unpredictable but definite states, whereas, the composite system remains in a pure-state superposition! The experiments demonstrate that the global MS is actually the collapsed state. Some experts (Sec. IV) argue that the fully collapsed incoherent mixture represented by the density operator,

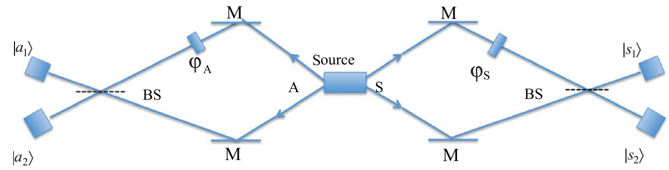

FIG. 1. (Color online) The RTO experiments. A source emits two entangled photons A and S into two paths with one path phase shifted. Mirrors M and BSs recombine the beams. Without entanglement, each photon would exhibit coherent interference as a function of φ_A or φ_S. Entanglement causes the photons to be in incoherent states having definite outcomes. The coherence is, however, preserved in the global measurement state.

$$(|a_1\rangle|s_1\rangle\langle s_1|\langle a_1| + |a_2\rangle|s_2\rangle\langle s_2|\langle a_2|)/2 \tag{3}$$

is the state we actually see upon measurement. They are contradicted by the fact that Eq. (3) has no entanglement and, thus, no nonlocal channel, whereas, the delayed-choice experiment [13] shows that measurement establishes a nonlocal channel. We will see that there is no contradiction between the MS Eq. (2) and the collapse postulate.

In the RTO experiment, a photon was "parametrically down-converted" into two entangled photons S and A, which moved through separate two-slit interference experiments and impacted separate detection screens. More accurately, the experiments used interferometers equipped with beam splitters (BSs), variable phase shifters, and photon detectors, but this is equivalent to two slits and a screen for each photon. The experiments were performed with photon pairs, but the results would be the same if they were performed with material quanta.

S and A emerged from the down-converter in the entangled "Bell state" Eq. (2) (Fig. 1), implying that each photon acted as an ideal von Neumann detector for the other photon. Each photon emerged from the down-converter in two superposed beams; call S's two beams $|s_i\rangle$ and A's two beams $|a_i\rangle$ $(i = 1,2)$. One beam of S was then phase shifted through an angle φ_S, and one beam of A was phase shifted through φ_A, analogous to observing off-center at S's and A's viewing screens in the equivalent two-slit experiment. S's two beams were then combined at a BS whose outputs were monitored by photon detectors and similarly for A's beams. See Refs. [19–21] and my conceptual physics textbook [22] for more on these experiments.

In an ensemble of two-photon trials, neither A's nor S's detector showed local (i.e., as actually observed at either detector) interference. This verified that each photon acted as a which-path detector for the other photon, decohering both photons and creating definite local outcomes, just like the one-photon two-slit experiment with a which-path detector at the slits. As observed locally, each photon impacted its detectors randomly with 50:50 probability regardless of φ_S

and φ_A. There was no sign of the local interference that would occur were S and A not entangled—a textbook example of measurement-induced decoherence.

But how then was unitary evolution preserved, despite local collapse and loss of coherence? The answer: RTO's nonlocal "fourth-order" (involving all four photon paths) pattern showed the coherence predicted by Eq. (2). A coincidence detector revealed coherence as a function of the phase difference $\varphi = \varphi_S - \varphi_A$ between S's phase angle and A's phase angle! The photons "knew" each others' phase angles, despite their arbitrarily large separation [22]. This certainly appears to be nonlocal, and indeed, the results violate Bell's inequality, verifying nonlocality [16]. "The entangled state of Eq. (2) shows a definite phase relation between the two-particle states, namely, $|s_1\rangle|a_1\rangle$ and $|s_2\rangle|a_2\rangle$ but no definite phase relations between single-particle states" [19] (the quotation uses the present paper's notation).

Special relativity implies that local incoherence *must* be the case because, otherwise, a change in φ_S would, by altering $\varphi = \varphi_S - \varphi_A$, instantly alter A's outcomes in a manner that A could detect locally even if A were in another galaxy. This would violate "Einstein causality:" S could send an instantaneous signal to A. Thus, φ must be "camouflaged" at both detectors, implying that neither photon shows interference [23,24]. So quantum entanglement delicately protects causality while permitting an instantaneous nonlocal channel between subsystems.

Thus, experiments confirm that, when SA is in the MS, the two-body states $|s_1\rangle|a_1\rangle$ and $|s_2\rangle|a_2\rangle$ are coherently superposed, whereas, the states of S and A are incoherently mixed. As we will see next, standard quantum physics (but with no collapse postulate) correctly predicts all of this.

III. THE MEASUREMENT STATE: THEORY

Einstein was perhaps the first to recognize nonlocality, although he used it to argue for an alternative view of quantum physics. During the 1927 Solvay Conference, Einstein noted that "a peculiar action-at-a-distance must be assumed to take place" when the Schrödinger field passes through a single slit, diffracts as a spherical wave, and strikes a detection screen. Theoretically, when the interaction localizes as a small flash at an isolated point on the screen, the field instantly vanishes over the rest of the screen. Supporting de Broglie's theory that supplemented the Schrödinger field with particles, Einstein commented "if one works only with Schrödinger waves, the interpretation of psi, I think, contradicts the postulate of relativity" [14], [25]. Since that time, however, the peaceful coexistence of quantum nonlocality and special relativity has been demonstrated [23,24].

It's well known today that Bell's theorem shows quantum physics contains a nonlocal element. Many experiments, culminating in those of Aspect [26,27], show (subject to only a few remaining highly speculative loopholes) that the quantum predictions agree with nature; moreover, nature is nonlocal even if it should turn out that quantum physics is wrong. The nonlocality of the entangled MS has been verified directly not only in RTO's two-photon interferometry experiment where both subsystems are photons, but also in the single-photon two-slit delayed-choice experiment where the second subsystem is a which-slit detector that can be quickly and randomly inserted [13].

S and A could be separated by light years (the current distance record for nonlocality demonstrations is 147 km [28]), but random changes in S's phase shifter are, nevertheless, predicted to show up in A's data with no time delay. We must take this nonlocality into account when analyzing the MS; even though S and A might be next to each other in an actual experiment, they *could* be widely separated without changing any essentials of the experiment.

Consider a series of trials of the RTO experiment in which each trial consists of a quick random change in S's phase shifter and the recording of data in both S's and A's detectors. Neither an observer at S nor an observer at A necessarily has any direct experimental knowledge of coincidences because the two could be light years apart. The only directly observed data are local. Global data are available only later.

Haroche and Raimond [29] have weighed in on this question of when a two-part system should be considered a composite of two subsystems. They advise us that a system should be considered single whenever the binding between its parts is much stronger than the interactions involved in its dynamics. By this criterion, a system and its measuring apparatus (which are not necessarily bound together at all) must be considered separate subsystems.

Theorem. When a bipartite system is in the MS, neither subsystem is in a local superposition.

Proof. Define the pure-state density operator $\rho_{SA} = |\Psi\rangle_{SA\,SA}\langle\Psi|$. As is well known, the expectation value of any observable Q of SA can be found from $\langle Q\rangle = \mathrm{Tr}(\rho_{SA} Q)$. For any observable Q_S operating on S alone, the MS implies

$$\begin{aligned}\langle Q_S\rangle &= \mathrm{Tr}_{SA}(\rho_{SA} Q_S) = \mathrm{Tr}_S(Q_S)\\ &= \langle s_1|Q_S|s_1\rangle + \langle s_2|Q_S|s_2\rangle,\end{aligned} \tag{4}$$

because

$$\mathrm{Tr}_A \rho_{SA} = \Sigma_k \langle a_k|\rho_{SA}|a_k\rangle = |s_1\rangle\langle s_1| + |s_2\rangle\langle s_2| = I_S \tag{5}$$

(the identity operator acting in S's Hilbert space). If S was in a local superposition $\beta|s_1\rangle + \gamma|s_2\rangle$ (β and γ are nonzero), the expectation of the observables $Q_S = |s_1\rangle\langle s_2| + |s_2\rangle\langle s_1|$ and $P_S = i|s_1\rangle\langle s_2| - i|s_2\rangle\langle s_1|$ would be $2\,Re(\beta\gamma*)$ and $2\,Im(\beta\gamma*)$, respectively. Now, $2\,Re(\beta\gamma*)$ is zero only if amplitudes β and γ are 90° out of phase, and in this case, $2\,Im(\beta\gamma*)$ is nonzero, so at least one of these two observables has a nonzero expectation. But Eq. (4) predicts zero for the expectations of both Q_S and P_S—a contradiction. Thus, S cannot be in a local superposition. The same argument goes for A. ■

Schrödinger's cat provides an example. Schrödinger famously considered a cat, a radioactive nucleus, and a device that automatically releases poison gas, killing the cat when the nucleus decays. If the nucleus S was isolated, then its state at a time of one half-life would be given by Eq. (1) where $|s_1\rangle$ and $|s_2\rangle$ are the undecayed and decayed states, respectively. But the nucleus + cat composite system is described by the MS, with A being the cat and $|a_1\rangle$, $|a_2\rangle$ representing its alive and dead states, respectively. Thus, the theorem says that Schrödinger's cat is not in a superposition $\beta|a_1\rangle + \gamma|a_2\rangle$ (β and γ are nonzero) of dead and alive states and similarly for the nucleus. Neither an observer at A nor an observer at S sees a superposition. So an observer at A must see either an alive or a dead cat, an observer at S must see an undecayed or a decayed nucleus, and a global observer sees the expected correlations between these outcomes.

This solves the problem of outcomes, at least for ideal measurements: The MS implies that observers at S and A find definite states $|s_i\rangle$ and $|a_i\rangle$, respectively, not superpositions. This prediction, based on standard quantum physics (however, with a collapse that is not postulated but is, instead, derived), is experimentally confirmed by the RTO experiments.

But the MS is a pure-state superposition. It shows a nonlocal relationship between the phases of the two separated photons. As experiment (Sec. II) and theory [19] show, photon pairs exhibit a sinusoidal dependence on the phase difference $\varphi = \varphi_S - \varphi_A$ between the individual photon phases φ_S and φ_A. This nonlocal phase relationship implies the coherence of the global MS.

The decoherence concept [2] is useful here. In the standard definition, decoherence refers to a loss of coherence among the terms of a superposition caused by measurementlike (i.e., entangling) interactions with the wider environment. But the decoherence can equally refer to the loss of coherence caused by laboratory measurements. In both the environmental and the laboratory cases, entanglement turns coherent superpositions into incoherent mixtures. In the case of an ideal measurement, decoherence occurs instantly when the apparatus and the observed system form the MS. This is carefully examined in the delayed-choice experiment [13] and is found to be truly nonlocal and instantaneous (or at any rate "faster than light").

To my knowledge, Rinner and Werner [7] were the first to publish a clear statement that the MS directly implies the observed collapse, although others [8,9] have suggested this result.

These results imply a refined understanding of the standard eigenstate principle according to which the state immediately following a measurement is an eigenstate of the measured observable. Because of the bipartite entangled nature of the MS, one must ask "which state: the global MS, or the local state?" Clearly, the global state, Eq. (2), following a measurement is not an eigenstate of the measured observable. In the eigenstate principle, the word state needs to be understood as the local state, or the "state that is actually observed." Thus, the standard quantum principles (minus the collapse postulate) imply that the observed states of quantum systems evolve by two mechanisms: the unitary Schrödinger evolution, punctuated by instantaneous collapse into a new state upon entanglement.

IV. ASSESSMENT OF PREVIOUS ANALYSES

There are widespread claims that Schrödinger's cat is not in a definite alive or dead state but is, instead, in a superposition of the two. van Kampen, for example, writes "The whole system is in a superposition of two states: one in which no decay has occurred and...one in which it has occurred. Hence, the state of the cat also consists of a superposition... $|\text{cat}\rangle = a|\text{life}\rangle + b|\text{death}\rangle$....The state remains a superposition until an observer looks at the cat" [30].

Bell [31], referring to van Kampen's description, agrees, stating that "After the measurement the measuring instrument, according to the Schrödinger equation, will admittedly be in a superposition of different readings...$|\text{cat}\rangle = a|\text{life}\rangle + b|\text{death}\rangle$." Many textbooks make the same assessment [32]. In fact, "Cat states have now come to refer to any quantum superposition of macroscopically distinct states" [33]. But Sec. III rigorously proved the MS to be inconsistent with superpositions of the individual subsystems. This invalidates the above claims.

The formulation $|\text{cat}\rangle = a|\text{life}\rangle + b|\text{death}\rangle$ obscures the nonlocal entanglement. The cat state must be written as $|\text{cat}\rangle = a|\text{live cat}\rangle|\text{undecayed nucleus}\rangle + b|\text{dead cat}\rangle|\text{decayed nucleus}\rangle$. This entangled state actually is the collapsed state of both the cat and the nucleus, showing definite outcomes. Contrary to van Kampen's and some others' opinions, "looking" at the outcome changes nothing, beyond informing the observer of what has already happened. Entanglement is the formal representation of looking. Once the composite system is in the MS, the looking has already occurred.

In an influential paper, Wigner derives the pure state Eq. (2) from the standard measurement analysis and contrasts it with Eq. (3), which the collapse postulate seems to imply [34]. The predictions of Eqs. (2) and (3) are nearly identical. Their local predictions for S and A are identical, and both predict global correlations such that $|s_i\rangle$ occurs if and only if $|a_i\rangle$ occurs. The only physical difference is that Eq. (2) is entangled and, thus, predicts a nonlocal channel that Eq. (3) does not predict. And on this count, Eq. (3) *cannot* represent the state we actually observe following a measurement because the delayed-choice two-slit experiment [13] shows that, following a measurement, there is, indeed, a nonlocal channel between S and A. Wigner suggests that the measurement problem would be solved if Eq. (3) could be derived using only the unitary Schrödinger evolution. He then proves that this is impossible, concluding that the equations of quantum physics do not predict definite outcomes. But we have seen that Eq. (2) does, in fact, imply definite observed outcomes.

Adler [35] states that "the quantum measurement problem consists in the observation that Eq. (2) is *not* what is observed as the outcome of a measurement! What is seen is not the superposition of Eq. (2), but rather *either* the unit normalized state $[|s_1\rangle|a_1\rangle]$ *or* the unit normalized state $[|s_2\rangle|a_2\rangle]$." Adler's "either/or" phrase is equivalent to Eq. (3). Similarly, Bassi and Ghirardi [36] derive Eq. (2) as a consequence of quantum physics and then claim that Eq. (2) contradicts the measurement postulate. We find this to be untrue.

V. CONCLUSIONS

When an apparatus A performs an ideal measurement that distinguishes between the terms of a superposed system S, the composite system SA evolves into the entangled MS, Eq. (2). Experimental and theoretical analyses of this state show it to be the locally collapsed state of both S and A. Despite these incoherent local states, the global composite state maintains its unitary evolution with the coherence of the original superposition now transferred to this global superposition. This MS exhibits, both experimentally and theoretically, a coherent nonlocal relationship between the phases of the two separated (perhaps widely separated) photons: The paired photons show a sinusoidal dependence on the difference $\varphi = \varphi_S - \varphi_A$ between their individual phases φ_S and φ_A. This transfer of coherence from the local to the global state is a consequence of unitarity and is surely fundamental to understanding the second law of thermodynamics in terms of quantum physics.

In short, for ideal measurements both experiment and standard quantum theory imply an instantaneous collapse to unpredictable but definite outcomes. The MS is the global form of the collapsed state with no need for a separate "process 1" or measurement postulate [3]. The other postulates of quantum physics *imply* that, when systems become entangled, their observed states instantly collapse into unpredictable but definite outcomes. In particular, Schrödinger's cat is in a nonparadoxical definite state, alive when the nucleus does not decay and dead when it does. This solution of the problem of outcomes requires no assistance from other worlds, human minds, hidden variables, collapse mechanisms, collapse postulates, or "for all practical purposes" arguments.

ACKNOWLEDGMENTS

S. L. Adler, M. Sassoli de Bianchi, R. Brooks, J. Gea-Banacloche, A. J. Leggett, M. Lieber, D. Mermin, P. Milonni, P. Pearle, S. Rinner, M. Schlosshauer, M. O. Scully, S. Singh, and R. Vyas provided useful exchanges. A reviewer provided important clarification.

[1] S. Weinberg, Phys. Rev. A **85**, 062116 (2012).
[2] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition* (Springer, Berlin, 2007).
[3] J. von Neumann, *The Mathematical Foundations of Quantum Mechanics*, (Princeton University Press, Princeton, 1955), translated by R. T. Beyer from the 1932 German edition, Chaps. 5 and 6.
[4] A. J. Leggett, Science **307**, 871 (2005).
[5] W. H. Zurek, Phys. Today **44**, 36 (1991); Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982); Prog. Theor. Phys. **89**, 281 (1993); Phys. Rev. A **76**, 052110 (2007).
[6] N. Gisin, Phys. Lett. A **154**, 201 (1991).
[7] S. Rinner and E. K. Werner, Central Eur. J. Phys. **6**, 178 (2008).
[8] M. O. Scully, R. Shea, and J. D. McCullen, Phys. Rep. **43**, 485 (1978).
[9] M. O. Scully, B. G. Englert, and J. Schwinger, Phys. Rev. A **40**, 1775 (1989).
[10] D. Dieks, Phys. Lett. A **142**, 439 (1989).
[11] D. Dieks, Phys. Rev. A **49**, 2290 (1994).
[12] O. Lombardi and D. Dieks, *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/qm-modal/.
[13] V. Jacques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect, and J.-F. Roch, Science **315**, 966 (2007).
[14] A. Hobson, Am. J. Phys. **81**, 211 (2013).
[15] F. Kaiser, T. Coudreau, P. Milman, D. B. Ostrowsky, and S. Tanzilli, Science **338**, 637 (2012); A. Peruzzo, P. Shadbolt, N. Brunner, S. Popescu, and J. L. O'Brien, *ibid.* **338**, 634 (2012).
[16] J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990).
[17] J. G. Rarity and P. R. Tapster, Phys. Rev. A **41**, 5139 (1990).
[18] Z. Y. Ou, X. Y. Zou, L. J. Wang, and L. Mandel, Phys. Rev. Lett. **65**, 321 (1990).
[19] M. A. Horne, A. Shimony, and A. Zeilinger, *Sixty-Two Years of Uncertainty* (Plenum, New York, 1990), p. 113.
[20] H. J. Bernstein, D. M. Greenberger, M. A. Horne, and A. Zeilinger, Phys. Rev. A **47**, 78 (1993).
[21] K. Gottfried, Am. J. Phys. **68**, 143 (2000).
[22] A. Hobson, *Physics: Concepts & Connections*, 5th ed. (Pearson Addison-Wesley, San Francisco, 2010), Chap. 13.
[23] L. E. Ballentine and J. P. Jarrett, Am. J. Phys. **55**, 696 (1987).
[24] P. H. Eberhard and P. R. Ross, Found. Phys. Lett. **2**, 127 (1989).
[25] M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley-Interscience, New York, 1974), p. 115.
[26] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[27] A. Aspect, Nature (London) **398**, 189 (1999).
[28] M. Lucibella, APS News **22**, 6 (2013).
[29] S. Haroche and J. M. Raimond, *Exploring the Quantum: Atoms, Cavities, and Photons* (Oxford University Press, Oxford, 2006), p. 52.
[30] N. G. van Kampen, Physica A **153**, 97 (1988).
[31] J. Bell, Phys. World **3**, 33 (1990).
[32] L. Ballentine, *Quantum Mechanics: A Modern Development* (World Scientific, Singapore, 1998); A. Galindo and P. Pascual, *Mecanica Quantica I* (Springer, Berlin, 1990), English language version; A. Goswami, *Quantum Mechanics* (Brown, Dubuque, IA, 1992); W. Greiner, *Quantenmechanik. 1, Einfuehrung* (Springer, Berlin, 1994), English language version; D. Griffiths, *Introduction to Quantum Mechanics* (Pearson/Prentice Hall, Upper Saddle River, NJ, 2005); A. Rae, *Quantum Mechanics* (Institute of Physics, Bristol, UK, 2002); R. Scherrer, *Quantum Mechanics: An Accessible Introduction* (Pearson/Addison-Wesley, San Francisco, 2006).
[33] J. Dunningham, A. Rau, and K. Burnett, Science **307**, 872 (2005).
[34] E. Wigner, Am. J. Phys. **31**, 6 (1963).
[35] S. L. Adler, Stud. Hist. Philos. Mod. Phys. **34**, 135 (2003).
[36] A. Bassi and G. C. Ghirardi, Phys. Rep. **379**, 257 (2003).