



Arkansas Water Resources Center

OPTIMAL GROUNDWATER MINING METHODS

PREPARED FOR:

WINTHROP ROCKEFELLER FOUNDATION
AND
INTERNATIONAL AGRICULTURAL PROGRAMS OFFICE
(VIA USAID TITLE XII PROGRAM)

PREPARED BY:

WATER RESOURCES MANAGEMENT LABORATORY
DEPARTMENT OF AGRICULTURAL ENGINEERING
RICHARD C. PERALTA
AND
KRZYSZTOF G. KOWALSKI

DECEMBER 1986

MSC-38

Arkansas Water Resources Center
112 Ozark Hall
University of Arkansas
Fayetteville, Arkansas 72701

OPTIMAL GROUNDWATER MINING METHODS

PREPARED FOR:
WINTHROP ROCKEFELLER FOUNDATION
AND
INTERNATIONAL AGRICULTURAL PROGRAMS OFFICE
(VIA USAID TITLE XII PROGRAM)

PREPARED
WATER RESOURCES MANAGEMENT LABORATORY
DEPARTMENT OF AGRICULTURAL ENGINEERING
RICHARD C. PERALTA
AND
KRZYSZTOF G. KOWALSKI

ARKANSAS WATER RESOURCES RESEARCH CENTER
UNIVERSITY OF ARKANSAS
223 OZARK HALL
FAYETTEVILLE, ARKANSAS 72701

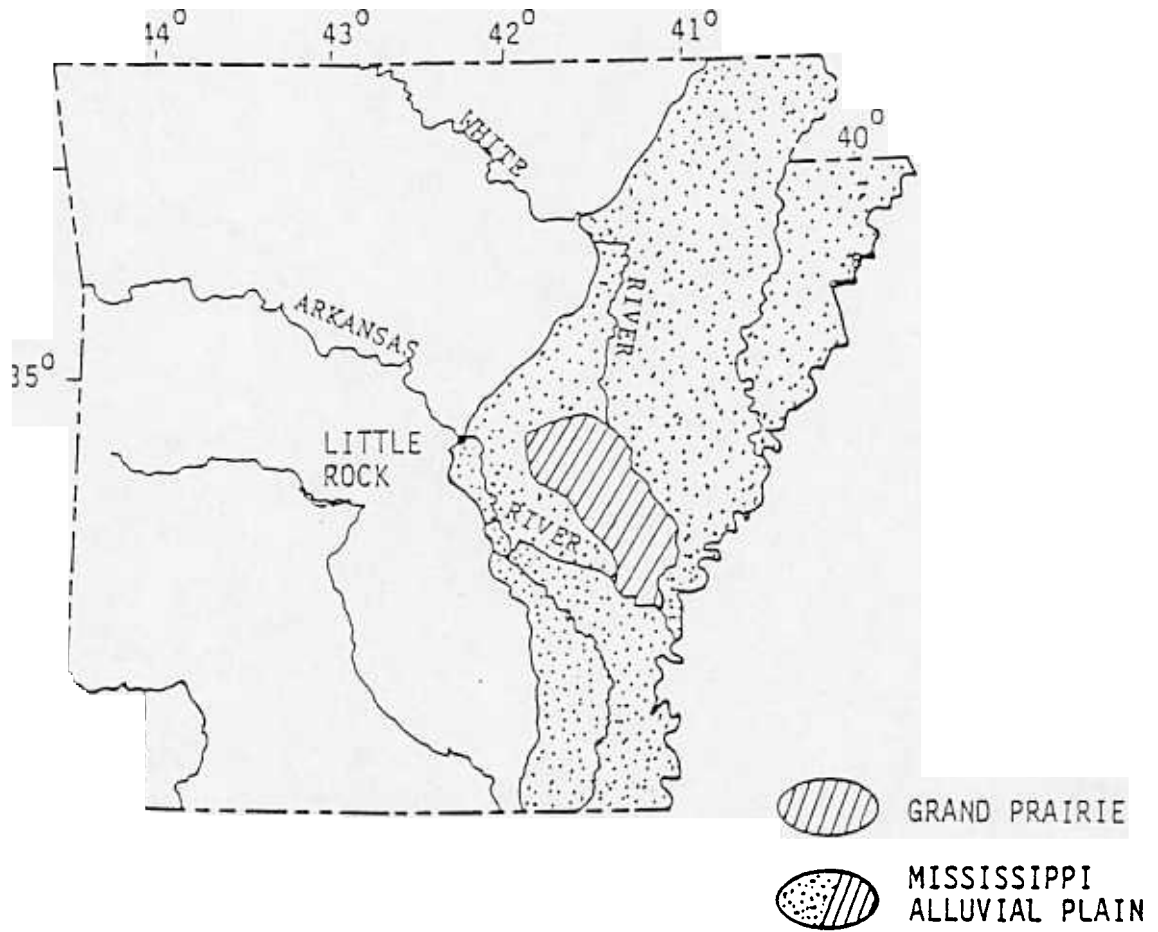
MISCELLANEOUS PUBLICATION No. 38
DECEMBER

INTRODUCTION

Via recent legislation, the Arkansas Soil and Water Conservation Commission is tasked with identifying critical groundwater areas for which increased control of groundwater extraction is necessary. Because of intensive use of groundwater for irrigation, the 4700 km² (1800 mile²) Grand Prairie (Fig. 1) is identified within the state water plan as a likely first area for such management. State and federal agencies are cooperating in planning for diversion of water to the region from nearby rivers to reduce agricultural reliance on groundwater. In addition, a local water district now exists to administer distribution of future diverted water. The construction implementation of such a diversion system is anticipated to require about 10 years. The presented study describes how to determine groundwater pumping strategies that are optimal for this time period and do not cause a disruption of groundwater flow patterns outside the Grand Prairie.

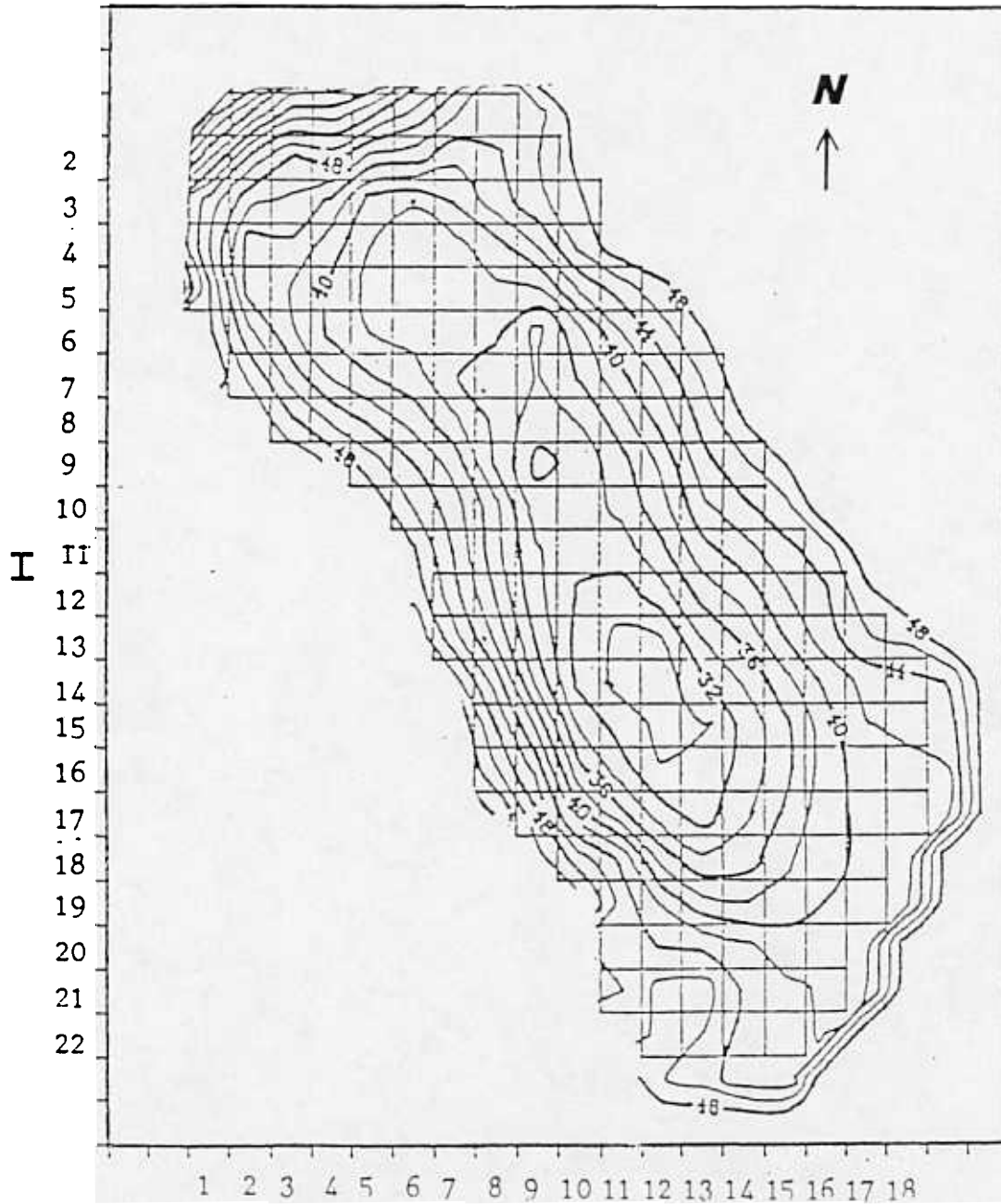
Groundwater is the primary source of water for irrigation of rice and soybeans in the agriculturally important Grand Prairie. A shallow alluvial aquifer, part of the Mississippi Plain alluvial aquifer, underlies the region. The aquifer is overlain by a relatively impermeable clay which makes the region ideal flood irrigated rice production, but prevents appreciable recharge via deep percolation. As a result of extensive groundwater use, water levels have been dropping for much of this century in the central portion of this area. Saturated thickness is dangerously thin. Figure 2 shows the degree to which

Fig. 1 Arkansas, the Grand Prairie and the Mississippi Plain
Alluvial Aquifer



Spring 1982 Groundwater Elevations in the Grand Prairie,

(m above sea level)



potentiometric surface is depressed.

Through simulation, Peralta et al (1985) determined that 7.6 m. (25 ft) was the minimum economically desirable springtime saturated thickness for a representative 1893 L/min (500 gpm well) pumping to provide irrigation water for 20 ha (50 acres) of rice in the Grand Prairie under average hydrologic conditions. They also judged that 6 m (20 ft) was the minimum springtime saturated thickness at which the representative well could physically yield adequate water throughout an average climatic irrigation season.

In 1983 there were approximately 140 km^2 (54 mile^2) in which the underlying saturated thickness was less than 7.6 m. A continuation of current extraction rates will result in saturated thickness of less than 7.6 m in about 350 km^2 (135 mile^2) by 1993 (Peralta et al, 1985). In addition, groundwater levels along that part of the periphery of the Prairie that is not recharged by streams may decline.

The first objective of this paper is to determine the maximum volume of groundwater that can be extracted by 1993 without causing peripheral water table elevations to decline (i.e., without violating assumed constraints on recharge entering the area from extensions of the aquifer outside the area). The second objective is to develop a pumping strategy that maximizes net economic return resulting from irrigation supported by groundwater during that planning period, subject to the same constraints. The final objective is to demonstrate the sensitivity of each strategy to the imposition of a constraint on

final saturated thickness (≥ 6 m) and the forcing of pumping to be unidirectional in change with time. A 'unidirectional' constraint prevents pumping from increasing in a cell after it has decreased in that cell.

PREVIOUS WORK AND ITS RELATION TO THE CURRENT STUDY

Gorelick (1983) provided a review of published works referencing use of the response matrix approach to groundwater simulation. A response matrix is comprised of linear influence coefficients that describe the response of the potentiometric surface to a unit volume of extraction or injection of groundwater. These coefficients, Dirac delta functions, (Maddock, 1972; Haimes and Dreizin, 1977) are also termed discrete kernels (Morel-Seytoux and Daly, 1975; Ilangasekare et al, 1984) or response values (Heidari, 1982; Danskin and Gorelick, 1985).

Some researchers used well-influence coefficients developed through the use of the Theis equation, (Maddock, 1972; Morel-Seytoux and Daly, 1975; Haimes and Dreizin, 1977; Heidari, 1982; Colarullo et al, 1984). Others utilized cell-influence coefficients developed via the Boussinesq equation (Haimes and Dreizin, 1977; Ilangasekare et al, 1984, Peralta and Kowalski, 1986).

Several researchers have demonstrated incorporation of the response matrix approach with optimization to develop optimal extraction strategies. Objective functions that have been utilized include: maximization of present value of net economic return (Maddock and Haimes, 1975; Haimes and Dreizin, 1977;

Colarullo et al, 1984), maximization of extraction (Heidari 1982), minimization of cost of supplying needed water (Danskin and Gorelick, 1985), and maximization of target potentiometric surface attainment (Peralta and Kowalski, 1986). Among these, Heidari (1982) and Danskin and Gorelick (1985) addressed problems of specific areas in Kansas and California respectively. Other referenced papers dealt solely with hypothetical areas.

This paper demonstrates application of the response matrix approach to develop optimal strategies for mining groundwater in the Grand Prairie region in Arkansas. This area is significantly larger than any other real area to which application of optimal response matrix methods has been reported. In addition, because of the large number of wells in the area, over 1000, this paper uses cell influence coefficients rather than well coefficients. Although the objective functions used in this study are basically the same as those used by Maddock and Haines (1975) and Heidari (1982), application to the Grand Prairie requires somewhat different constraints. The differences are summarized here, although specific formulations are found subsequently in text.

The first modification arises because the Grand Prairie is part of an extensive aquifer system. Thus, constraints that limit the simulated flow into the area from extensions of the aquifer outside the area are formulated and included in the presented management model. In his study, Heidari (1982) studied a fairly isolated aquifer system. In that study, recharge constraints were not intrinsically imbedded as flow equations within the model. Instead, recharge was considered in establishing upper limits on

pumping at the well fields.

A second difference between this work and that of previous studies that have optimized extraction strategies is that the presented model is applied to an initially hydraulically stressed potentiometric surface--one that is not at steady-state. convolution equation previous optimizers have used assumed that the potentiometric surface was initially at approximately steady-state conditions and that the initial water level would remain as it was if no stimulus occurred. In the current paper, we utilize the convolution equation presented for simulation purposes by Morel-Seytoux et al (1981) and Illangasekare et al (1984). This formulation includes consideration of the fact that the initial ground water levels may not be steady. Thus if no hydraulic stimulus occurred the system would gradually relax, as it would in nature.

This study also provides a comparison of the consequences of maximizing pumping versus maximizing net economic return (NER). This includes demonstration of the sensitivities of the optimal strategies to constraints on water levels and the manner in which pumping can vary with time.

THE MANAGEMENT MODEL

Objective functions used in models in this study are those that maximize total groundwater extraction, G , and maximize the total present value of net economic return resulting groundwater extraction, NER. Most simply, these are of following forms for the respective strategies:

$$\max G = \sum_{k=1}^K \sum_{i=1}^J g_{i,k} \quad \dots\dots 1$$

$$\max \text{NER} = \sum_{k=1}^K \sum_{i=1}^J c'_k g_{i,k} - c''_k (h_i - h_i^o + h_i^d + s_{i,k}) g_{i,k} - c'''_k g_{i,k} \quad \dots\dots 2$$

where

K is the number of time steps in the planning period;

J is the number of variable-head cells in the study area;

c'_k is a coefficient representing the present value of the net return occurring in time step k resulting from irrigation using a unit volume of groundwater, excluding the cost of supplying water, ($\$/L$);

c''_k is a coefficient representing the present value of the cost of lifting a unit volume of groundwater one unit distance in time step k, ($\$/L$). It includes energy, repair and lubrication costs for the pumping power plant;

c'''_k is a coefficient representing the present value of the pump maintenance costs of pumping a unit volume of groundwater in time step k, ($\$/L$).

h_i^g is the ground surface elevation in cell i, (L);

h_i^o is the initial potentiometric surface elevation in cell i, (L);

$h_{i,k}^d$ is the average seasonal dynamic drawdown expected at a

representative pumping well in cell i at time step k, (L);

$s_{i,k}$ is the difference in groundwater level at the center of cell i between the initial level and the level at the end of time step k, (L). It is a positive valued drawdown if the level has declined.

$g_{i,k}$ is the groundwater that is extracted from the aquifer and used for irrigation in cell i in time step k, (L).

The model requires the use of bounds and constraints to assure that physical and institutional limits are appropriately considered and that the hydrologic system is modelled adequately. Assuming discharge to be positive in sign and recharge to be negative, these are:

$$0 \leq g_{i,k} \leq w_i \quad \text{for } i = 1 \dots J, k = 1 \dots K \quad \dots 3$$

$$s_{i,k} \leq s_{i,k}^U \quad \text{for } i = 1 \dots J, k = 1 \dots K \quad \dots 4$$

$$e_{l,k} \leq e_{l,k}^L \leq e_{l,k}^U \quad \text{for } l = 1 \dots L, k = 1 \dots K \quad \dots 5$$

and, if it is desirable that the annual pumping volume in a cell not increase after it has decreased from current pumping (unidirectional change):

$$g_{i,k+1} \leq g_{i,k} \quad \text{for } i = 1 \dots J, k = 1 \dots K-1 \quad \dots 6$$

where

w_i is the volume of groundwater required for irrigation to support current (1982) acreages in cell i under

average climatic conditions in a single time step (L);

$s_{i,k}^U$ is the upper bound on acceptable drawdown in cell i by the end of period k, (L);

$e_{l,k}$ is the volume of groundwater that will enter the study area aquifer in peripheral cell l and time step k from extensions of the aquifer outside the study area, (L);

$e_{l,k}^L$ and $e_{l,k}^U$ are lower and upper bounds on the volume of groundwater flowing between the aquifer underlying cell l and extensions of the aquifer outside the study area in time step k, (L);

L is the number of peripheral cells surrounding the variable-head cells of the study area. In this study all peripheral cells are constant-head/restrained flux cells.

In order to minimize computer storage requirements, neither $s_{i,k}$ nor $e_{i,k}$ are explicitly used as variables within the models. Instead, they are represented as algebraic technological functions in the following way. First, adopting the convolution equation described by Morel-Seytoux et al (1981) and Illangasekare et al (1984), the change in water level in cell i by the end of time period N is:

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \{ B_{i,j,N-k+1} (q_{j,k} - q_{j,k}^{ass}) \}$$

.....7

where

$B_{i,j,N-k+1}$ is a nonnegative-valued linear influence coefficient that describes the effect on the hydraulic head at cell i in time step N caused by $(q_{j,k} - q_{j,ass})$. The temporal subscript $N-k+1$ is used merely to insure that the proper B is utilized in each time step, (T/L) ;

$q_{j,k}$ is the net vertical hydraulic stimulus in cell j in time step k . It is the sum of all vertical discharges from the aquifer and recharges to the aquifer from the ground surface, (L/T) ;

$q_{j,ass}$ is the net vertical hydraulic stimulus that must occur in each time step in cell j for that cell to maintain its initial head. It is calculable using the linearized Boussinesq equation for steady-state two-dimensional flow through porous media and does not necessarily represent a steady-state stimulus that is actually occurring initially, (L/T) ;

Assuming that there is no stream/aquifer interaction inside the Grand Prairie and that there is negligible deep percolation entering the aquifer in that region (Peralta et al. 1985), $q_{j,k}$ in equation 7 can be replaced with $g_{j,k}$. Replacing the left hand side (LHS) of equation 4 with the modified right hand side (RHS) of equation 7, yields a constraint equation expressed in terms of decision variables, g , and knowns. The right hand side of equation 4 is the maximum drawdown that will provide an adequate predetermined springtime saturated thickness in each cell. Assumed to be known a priori, it may be based on hydraulic,

economic or legal criteria.

Substitutions can similarly be made to explicitly express equation 5 for each peripheral constant-head/restrained-flux (CH/RF) cell in terms of unknown pumping values and knowns. For a CH/RF cell there is no change in storage. Thus $e_{1,k}^U$ equals the maximum acceptable sum of groundwater flowing from cell 1 to adjacent CH/RF cells and internal variable-head (VH) cells. Net flow between CH/RF cells is easily determinable using Darcy's Law. Let $e'_{1,k}$ be defined as the net flow between CH/RF cell 1 and all adjacent VH cells. Using Darcy's Law for square cells, we can rewrite the right-hand two-thirds of equation 5 for a specific CH/RF cell 1 and time step N.

$$\sum_{i=1}^I \left\{ (h_{1,c} - h_{i,o} + s_{i,N}) \sqrt{(T_1)(T_i)} \right\} \leq e'_{1,N} \quad \dots\dots 8$$

where

I is the number of variable-head cells adjacent to constant-head/restrained-flux cell 1;

$h_{1,c}$ is the constant head in cell 1, (L);

$\sqrt{(T_1)(T_i)}$ is the geometric mean transmissivity between cells 1 and i, (L/T).

Replacing $s_{i,N}$ with the RHS of equation 7 and rearranging to get all known values on the right yields:

$$\sum_{i=1}^I \sqrt{T T_{l i}} \sum_{k=1}^N \sum_{j=1}^J B_{i,j,N-k+1} g_{j,k} \leq$$

$$e'_{1,N} - \sum_{i=1}^I \sqrt{T T_{l i}} (h_{1,l}^c - h_{1,i}^o - \sum_{k=1}^N \sum_{j=1}^J B_{i,j,N-k+1} q_{j,ass}^a)$$

for $l = 1 \dots L, k = 1 \dots K$ 9

There should be one equation 8 for each CH/RF cell and each time step. However, in order to reduce computer memory requirements, equation 9 can be converted into a variation of constraint equation 4 for those CH/RF cells which are adjacent to no more than one VH cell. To do this the recharge constraint on cell l is converted into a value of s_U for the neighboring internal cell i . Then the value of s that is ultimately used for cell i is the lesser of: 1) the predetermined maximum drawdown that will leave adequate springtime saturated thickness for primarily economic or legal reasons (direct use of equation 4), or 2) the value calculated by the procedure and equation 11 described below.

For the stated case, all of the groundwater flowing from CH/RF cell l to VH cells flows to a single cell i . Again, the upper bound on this water, e'_1 , equals the difference between the maximum acceptable volume that can enter the aquifer in cell l from extensions of the aquifer outside the study area, e_U , and the net flow between cell l and any adjacent CH/RF cells. Since e_U is assumed known, the flow between CH/RF cells is calculable, and e'_1 is easily determined. From Darcy's law this maximum acceptable influx into cell i from cell l occurs when the head in

cell i is $h_{i,N}^{\min}$.

$$e'_{1,N} = (h_{1,N}^c - h_{i,N}^{\min}) \sqrt{(T_1)(T_i)} \quad \dots 10$$

where

$h_{i,N}^{\min}$ is the minimum acceptable head in cell i that will not violate the recharge constraint (equation 5) for adjacent CH/RF cell 1 in time step N. (L).

Solving eq. 10 for $h_{i,N}^{\min}$, realizing that $s_{i,N}^U = h_{i,N}^o - h_{i,N}^{\min}$, and substituting for $h_{i,N}^{\min}$ yields the desired variant of equation 4:

$$s_{i,N}^U \leq s_{i,N}^o = h_{i,N}^c - h_{i,N}^{\min} + (e'_{1,N} / \sqrt{T_1 T_i}) \quad \dots 11$$

Where once again, $s_{i,N}^U$ is calculated by the RHS of equation 7. As previously stated, in practice, the RHS of equations 4 and 11 are compared for those VH cells adjacent to a CH/RF cell that serves only a single VH cell. The lesser value of $s_{i,N}^U$ is the value selected for use in the model. The result of using this approach is to reduce the number of needed recharge constraints from L to (L - L1) where L1 is the number of CH/RF cells adjacent to only a single VH cell.

In summary, the models consist of one objective function (either equation 1 or 2); JxK variable pumping values bounded via equation 3; JxK equations (nos. 4 or 11) to limit the maximum acceptable cell drawdown to satisfy legal, economic and some

boundary recharge constraints; L-L1 of equation 9 to satisfy other boundary recharge constraints; and either none or $J_x(K-1)$ of equation 5, depending on whether the change in pumping is to be unidirectional.

APPLICATION AND RESULTS

The Grand Prairie overlies merely a portion of the Mississippi Alluvial aquifer, an extensive aquifer system of $42,500 \text{ km}^2$ ($16,400 \text{ mile}^2$) in Arkansas alone (Fig. 1). It would be preferable to be able to optimize the entire aquifer system, however this is computationally impractical. Computer memory requirements of optimization/simulation models are greater than those of pure simulation models. This fact (and economics) requires that optimization be performed on only a portion of the entire aquifer system, and so requires the assumption of conditions along boundaries that may not be hydrologic in nature.

We have chosen to assume that the Grand Prairie periphery can be treated as consisting of constant-head/restrained-flux cells. This is necessary because precise knowledge of boundary conditions (b.c.) is lacking. We know that the Mississippi Alluvial aquifer completely surrounds the Grand Prairie. Groundwater enters the Grand Prairie from extensions of the aquifer system via almost all sides (Note hydraulic gradients in Fig. 2). Common practice in simulating such situations is to model the boundaries with either constant-head (Dirichlet) or constant-flux (Neumann) conditions. Because our model combines the capabilities of both simulation and optimization, we can use

slightly different boundary conditions in the model. We treat each boundary cell as having constant head, but also prevent the recharge induced to enter the study area through that cell from the surrounding aquifer from exceeding some upper limit. In other words, we assume that constant-head boundary conditions are physically reasonable, as long as hydraulic gradients developed within the study area will not induce more than the 'maximum physically feasible recharge' which will maintain relatively 'constant' boundary elevations. Therefore, we assume an aquifer system comprised of internal variable-head cells surrounded entirely by constant-head/restrained-flux cells. Each cell in this study is the size of one-quarter of a township.

The use of constant-head/constrained-flux b.c. is preferred to the use of constant-flux b.c. for situations in which one is optimizing management in only a portion of a larger aquifer system. If one were to use constant-flux b.c. along the northern edge of the Grand Prairie region the model would force acceptance of the specified flux rate, whether acceptance enhanced objective attainment or not. Unless head is constrained in those constant-flux cells, one risks having the calculated boundary water levels increase in order to accept the specified flux rate. This is physically unrealistic. Furthermore, even if the water levels in those cells decrease, there is no assurance that such change will not cause unacceptable changes in the regional flow patterns. By using constant-head/constrained-flux b.c. one can permit the region to induce any rate that does not exceed a predetermined rate of acceptable recharge, without causing unacceptable head

changes. This approach is flexible since the limit on acceptable recharge may be based on either physical feasibility or legal right.

We also assume that the only discharges from the aquifer that can occur at internal cells are at pumping wells and that no recharge can occur at internal cells. A relatively impermeable clay layer exists between the ground surface and the alluvial aquifer. Recharge to the aquifer via deep percolation from the ground surface or streams is negligible (Griffis, 1972; Peralta et al, 1985).

The aquifer was confined prior to development. At the present time however, it is unconfined throughout the central portion of the study area and is probably unconfined in the vicinity of most wells during pumping. As Maddock (1974), Heidari (1982) and Danskin and Gorelick (1985) have pointed out, the use of influence coefficients that ignore changes in transmissivities may induce error in calculated water levels.

If knowledge of transmissivities is sufficiently accurate to require the action, sequential optimizations can be utilized to cause convergence to optimal solutions that accurately consider changing transmissivities. Danskin and Gorelick (1985) used this approach with influence coefficients. Peralta and Killian (1985) used the same approach of repetitive optimization with the embedding technique of optimizing sustained groundwater yield.

Knowledge of initial transmissivities in the Grand Prairie is insufficiently accurate to justify use of repetitive optimizations to correct for changing transmissivities during the planning period. Assuming that hydraulic conductivities are known

with absolute certainty, error in estimating transmissivity is proportional to error in estimating saturated thickness. In the Grand Prairie, the 95 % confidence interval on saturated thicknesses in the center of cells is about ± 6 m. Predicted saturated thicknesses that lie within the confidence interval cannot statistically be said to be different.

We assume that initially estimated transmissivities are adequate for the predictive purposes of this study. All saturated thicknesses resulting from optimal strategies reported in this paper are within the 95 % confidence intervals on estimated initial saturated thicknesses. In fact, changes in saturated thickness of all but 6 cells lie within the confidence intervals contributed by uncertainty in knowledge of water levels alone--even ignoring incomplete knowledge of aquifer base elevations.

Bounds, Constraints and Coefficients

The values used as w_i (upper bounds on pumping) in Equation 3 are the volumes of groundwater that are currently being withdrawn from the aquifer, based on 1982 crop acreages and average climatic conditions. It is assumed that water needs currently being satisfied by other sources will continue to be met by those sources.

Pumping is also bounded in some optimizations using Equation 6. This unidirectional constraint is practical for a situation in which a management agency is planning the gradual increase or decrease in acreages that can be irrigated with groundwater. Similarly, water users probably prefer to plan for either

increasing or decreasing irrigated acreages rather than for irregular increases interspersed with decreases. In our example, since current pumping is the initial upper bound on pumping, imposition of this constraint would promote the gradual decrease in pumping.

U

The values used as s (upper bounds on drawdown) in Equation 4 for most cells are those values that will leave at least either 3 or 6 meters (10 or 20 feet) of saturated thickness remaining at the end of each time step. For some optimizations, the 3 meter criterion is used. Three meters is appropriate because all saturated thicknesses predicted for 1993, if current pumping is continued, exceed that value (Peralta et al. 1985). Six meters is used for other optimizations. It is an estimated minimum springtime saturated thickness needed to insure adequate water for average climatic conditions (Peralta et al. 1985). The purpose of performing optimizations with both values is to assess the sensitivity of the solutions to this constraint.

For other cells, as explained previously, upper bounds on drawdown are determined by considering the maximum feasible recharge rates at adjacent boundary cells via Equation 11. Maximum feasible recharge rates are also used in the RHS of Equation 9.

The assumed physically feasible recharge rates at peripheral cells are the average of values observed based on springtime gradients between 1973 and 1983. If the Grand Prairie did not already have a stressed potentiometric surface, using historic recharge rates would be tantamount to overconstraining the problem. In effect, one would be preventing recharge from

increasing to its feasible limits. As Figure 2 illustrates, however, the surface is stressed. In fact, water levels have historically dropped somewhat even in those cells designated as being on the boundary. By using average springtime rates we assume that the historic drop in water levels in boundary cells is due to excessive stress induced by increased pumping during droughty conditions or by the steepened summer gradients resulting from pumping for irrigation. Using recharge rates based on springtime gradients is a compromise between overconstraining and underconstraining the problem.

Both models are run for a period of 10 years. The coefficients used in the objective function that maximizes the present value of groundwater withdrawal (Equation 2) are computed based on annual compounding using a $8 \frac{3}{8} \%$ discount factor. We assume that all groundwater that is pumped will be used to irrigate a crop mix that is one third rice and two thirds soybeans. For average climatic conditions and soil types, such a mix requires $3.21 \times 10^3 \text{ m}^3/\text{ha}$ (1.054 ac-ft/ac) of water per season. There is one crop season per year. The coefficients presented below are valid for the first year. Discounting is used to compute the coefficients for subsequent years.

Based on 1983 crop budgets (Smith et al, 1983; Stuart et al, 1983) the net return per unit volume of irrigation water not counting the cost of supplying water, c' , equals 9.708×10^{-2} $\$/\text{m}^3$ ($119.54 \text{ \$/ac-ft}$). The cost of lifting a unit volume of groundwater one unit distance, c'' , is 4.8×10^{-4} $\$/\text{m}^4$ ($0.18 \text{ \$/ac-}$

2
ft). The cost of pump maintenance costs, c'' , is 1.34×10^{-3} \$/m³ (1.65 \$/ac-ft). Distribution systems costs are ignored for purposes of this study.

The initial heads, h^o , are the heads observed in 1983. The average seasonal drawdowns at wells, h^d , are easily calculated based on saturated thicknesses extant during the planning period. Peralta et al (1985) show the relation between initial saturated thickness and average seasonal dynamic drawdown for a representative pumping well.

Influence coefficients are computed via a program by Verdin et al (1981) that uses the Boussinesq equation for unsteady flow. A 0.3 effective porosity, 82.3 m/day (270 f/day) hydraulic conductivity and spatially varied saturated thickness obtained from records of well construction are used (Engler et al, 1945; Sniegocki, 1964; Griffis, 1972; Peralta et al, 1985). Optimization is accomplished by the generalized differential algorithm in a subroutine prepared by Liefsson et al (1981).

Results and Discussion

Four different optimizations were performed for each of the two models (Eq. 1 and 2). The four optimizations represent possible combinations of: 1) constraining saturated thicknesses to be at least 6 m or at least 3 m, and 2) forcing pumping to be unidirectional in change with time or letting it change freely within initial bounds.

Table 1 summarizes the consequences of either continuing current groundwater pumping or implementing any of four maximum

Table 1. Consequences of Current Groundwater Extraction
and Maximum Groundwater Withdrawal Strategies

Strategy	Groundwater Extraction			Water Mining ^a %	Present Value ^b 10 \$
	6 m	3 m	10 m		
	First Five Years	Second Five Years	Total		
Current Policy	1734	1734	3468	≥ 48	
Unidirectional Pumping					
^b					
MST = 6 m	1047	979	2026	29	112
MST = 3 m	1086	1058	2144	33	117
Free Pumping					
MST = 6 m	993	1162	2155	33	116
MST = 3 m	1070	1230	2300	37	124

^a The mining percentage represents that portion of the total pumping that is not replaced by recharge.

^b MST is the minimum saturated thickness which is acceptable in a given strategy.

extraction strategies (Eq. 1). The imposition of constraints causes a reduction in the number of feasible solutions and often results in a reduction in objective attainment. For example, imposition of the unidirectional constraint causes a reduction in total pumping from that attainable by comparable strategies without that constraint. In addition, one notes that more water is pumped in the first five years than in the last five.

Scrutiny of the total pumping values in Table 1 shows an increase from top to bottom among the optimal strategies. This is expected since those strategies at the top are the most constrained. The top-most strategy does not permit pumping to increase with time after it has decreased, and allows water levels to drop no farther than 6 m above the aquifer base. This can be contrasted with the fourth optimal strategy that permits pumping to change freely with time, and allows water levels to drop to within 3 m of the base.

The most free optimal strategy permits total pumping of 2.3×10^9 m³. Constraining saturated thickness to be at least 6 meters instead of 3 m causes a reduction of 6 %. A management agency will probably wish to assure at least 6 m since the lowering of water levels which causes wells to become inoperable may result in litigation under Arkansas water law (Peralta et al, 1986). Imposition of the additional constraint of unidirectional change in pumping causes a cumulative reduction of 12 %.

Total pumping for the optimal strategies is between 2.03×10^8 and 2.30×10^8 m³ per year. These values are at least 33 % less than the average 3.47×10^8 m³ per year that are currently being

extracted. Clearly, if no constraints are placed on future pumping by state or local water managers, water levels in peripheral boundary cells, as well as in internal cells, can be expected to decline.

The validity of the scenario results are dependent on the degree to which the imposed hydrologic assumptions are correct. The optimization models numerically satisfy all imposed assumptions, but if the assumptions are overly or underly restrictive, the competitiveness of the strategies may be improperly limited or enhanced. For example, those strategies that allow water levels to approach within 3 m of the aquifer base assume that, despite a relatively thin saturated thickness, specified groundwater withdrawals can still be obtained from existing wells.

The policy of continuing current pumping also assumes that the pumped water can actually be obtained despite thinning saturated thicknesses and excessively high induced peripheral recharge rates. This 'do-nothing' management policy was tested using a simulation model that assumed constant-head boundary conditions, but could not constrain flux. Thus the results of this scenario assume that boundary heads can be maintained regardless of pumping. Since this is unlikely, the results from the policy of inaction are optimistic. They are presented in Table 1 merely to provide figures with which to compare the optimal strategies.

For each scenario Table 1 presents a mining percentage, the percentage of groundwater pumping that is not replaced by recharge during the ten year planning period. An indefinite value

is shown for the policy of continuing current pumping because of inexact knowledge of future declines in peripheral heads and their effect on groundwater levels. Among the optimal strategies, the increase in mining percentage with increasing pumping means an increase in groundwater extraction in the central portion of the region--cells distant from recharge sources.

The estimated present value of producing irrigated crops with all the pumped groundwater are also shown for all optimal scenarios. These values are computed, after optimization, using the appropriate water levels and the economic coefficients utilized for the NER model. Since there is an assumed net return for all water used for irrigation, projected return increases as pumping increases from left to right. An adverse effect of increasing pumping lift caused by declining water levels exists, but is relatively insignificant. The present value per unit volume of water ranges from $5.528 \times 10^{-2} \text{ }^3 \text{ } \$ / \text{ m}$ on the left to $5.391 \times 10^{-2} \text{ }^3 \text{ } \$ / \text{ m}$ for the fourth optimal strategy.

The most free optimal strategy results in a present value of 124 million dollars. Imposition of the 6 m constraint reduces the present value by 6 %. Addition of the unidirectional constraint causes a cumulative reduction of 10 %.

Table 2 compares maximum NER strategies with comparable maximum pumping strategies using percentages. Numbers in Table 2 are obtained by dividing values obtained from maximum NER strategies by values from comparable maximum pumping strategies.

Quick scanning reveals that the total pumping and total present value of the maximum pumping strategies are very similar

Table 2. Comparison between Strategies of Maximizing Present Value of Net
 Economic Return and Maximizing Extraction, expressed as a percent^a

Strategy	Groundwater Extraction			^b Water Mining	Present Value
	First Five Years	Second Five Years	Total		
Unidirectional Pumping					
^c MST = 6 m	101	99	100	99	100
MST = 3 m	101	98	100	98	100
Free Pumping					
MST = 6 m	105	94	99	98	100
MST = 3 m	102	98	100	100	100

^a 100 % times the value for the maximum net economic return strategy divided by the value for the maximum pumping strategy

^b The mining percentage represents that portion of the total pumping that is not replaced by recharge.

^c MST is the minimum saturated thickness which is acceptable in a given strategy.

to those of the maximum return strategies. Before rounding however, the total pumpings of the maximum pumping strategies are slightly greater than those of the the maximum return strategies. Similarly, the present value of the maximum net return strategies is slightly greater than that of the maximum pumping strategies.

One would expect a maximum NER strategy to pump about as much as a maximum pumping strategy since, in the example, economic return is generated only by pumping and using groundwater for irrigation. The fact that the maximum pumping strategy would generate as much economic return as a maximum NER strategy is less obvious. It is due to the fact that the economic gain derived from pumping early in the planning period is offset by the increase in costs caused by increased pumping lifts resulting from the early pumping.

A difference appears when one compares the temporal distribution of water use. Discounting serves to make pumping early in the period more valuable than pumping later in the period. Thus the percentage values shown in Table 2 exceed 100 percent for the first five years and are less than 100 percent for the second five.

In summarizing the results, one concludes that strategies developed using the maximum pumping and maximum NER models are volumetrically and fiscally comparable. The spatial distribution of the total pumping values obtained by each model are also very similar. Whether an agency selects one or the other model for use may depend on legislative or institutional mandates. After selecting the appropriate model, the agency must determine which strategy is most desirable.

When selecting a strategy, an agency needs to consider the effect of imposed constraints on the aquifer system and water users. As previously stated, the 6 m constraint is sound, based on Arkansas water law. On the other hand, the unidirectional constraint is probably not essential for management in the Grand Prairie. It may even be politically impractical. Although this constraint may be useful to water users attempting to systematically change acreages supported by groundwater, it also limits freedom. The 6 % reduction in pumping and 4 % reduction in present value may be too high a price for a dubious benefit. Most acceptable are the maximum pumping or maximum NER strategies which assure at least 6 m of saturated thickness while permitting pumping to vary freely with time.

CONCLUSIONS

We have demonstrated models for determining optimal groundwater extraction strategies that can exist for a specified planning period without violating the boundary conditions that must exist in any study area that is a subset of a larger aquifer system. This is important because it is often economically, computationally or physically impractical to attempt to optimize planning of an entire aquifer system at one time.

Strategies that maximize groundwater pumping are compared with those that maximize the present value of net economic return (NER) generated by pumping. In the presented examples, implementation of strategies for either objective would yield comparable results in terms of total pumping or economic return.

The gain in present value induced by pumping early in the 10-year planning period is offset by the increased pumping costs caused by increased pumping lifts resulting from the early pumping.

The least restricted maximum pumping and maximum NER strategies are developed using some bounds on pumping and water levels in all internal cells and bounds on induced recharge in peripheral cells. These permit pumping a total of $2.3 \times 10^9 \text{ m}^3$ in ten years and result in a present value of \$ 124 million.

Other strategies are developed which use different constraints on the resulting saturated thicknesses and on the direction of permitted changes in pumping with time in each cell. The most acceptable strategy differs from the least constrained strategy only in that it assures that at least 6 m of saturated thickness remains in each cell by the end of the planning period. This provides a degree of assurance that there will still be sufficient saturated thickness for representative pumping wells to supply their design discharge. Another benefit is the avoidance of litigation that can result when wells become inoperable due to insufficient saturated thickness.

The most acceptable strategy results in a total pumping of $2.155 \times 10^9 \text{ m}^3$, a reduction in 6 % from the least constrained strategy. The present value of this strategy is \$ 116 million, also a 6 % reduction.

Assume, for legal reasons, that only strategies that can assure at least 6 m of remaining saturated thickness after ten years are acceptable. In that case, the next most acceptable strategy, for systematic planning purposes, is one which adds the condition that pumping can never increase beyond the pumping

value of the previous year. Imposition of this constraint causes cumulative reductions of 12 % in pumping and 10 % in net return from the most free strategy. Since this constraint reduces the freedom of water users it may be politically unfeasible in some situations.

An agency seeking to maintain regional groundwater flow may utilize the techniques or information presented in this paper to set limits on the volume that can be extracted in each cell during a particular time period. For example, the study is useful to water planners seeking to solve a regional groundwater problem in Arkansas.

LITERATURE CITED

- Colarullo, S. J., Heidari, M. and Maddock, T. III. 1984. Identification of an optimal groundwater management strategy in a contaminated aquifer. *Water Resour. Bul.*, 20: 747-760.
- Danskin, W. R. and Gorelick, S. M. 1985. A policy evaluation tool: management of a multiaquifer system using controlled stream recharge. *Water Resour. Res.* 21: 1731-1747.
- Dreizin, Y. C., and Haines, Y. Y. 1977. A hierarchy of response functions for groundwater management. *Water Resour. Res.*, 13: 78-86.
- Engler, K., Thompson, D. and R. Kazman. 1945. Groundwater supplies for the rice irrigation in the Grand Prairie region, Arkansas. Bulletin No. 457, Agricultural Experiment Station, University of Arkansas, Fayetteville, AR.
- Gorelick, S. M. 1983. A review of distributed parameter groundwater management modeling methods. *Water Resour. Res.*, 19: 305-319.
- Griffis, C. L. 1972. Modelling a groundwater aquifer in the Grand Prairie of Arkansas. *Trans. ASAE*, 15: 261-263.
- Haines, Y. Y. and Dreizin, Y. C. 1977. Management of groundwater and surface water via decomposition. *Water Resour. Res.*, 13: 69-77.
- Heidari, M. 1982. Application of linear system's theory and linear programming to ground water management in Kansas. *Water Resour. Bul.*, 18: 1003-1012.

Illangasekare, T. H., Morel-Seytoux, H. J. and Verdin, K. L. 1984. A technique of reinitialization for efficient simulation of large aquifers using the discrete kernel approach. Water Resour. Res., 20: 1733-1742.

Liefsson, T., Morel-Seytoux, H. J. and Johch-Clausen, T. 1981. User's manual for QPTHOR: a FORTRAN IV quadratic programming routine. HYDROWAR Program. Colorado State University, Ft. Collins, Colorado. 70 p.

Maddock, T., III. 1972. Algebraic technological function from a simulation model. Water Resour. Res., 8: 129-134.

Maddock, T., III. 1974. Nonlinear technological functions for aquifers whose transmissivities vary with drawdown. Water Resour. Res., 10: 877-881.

Maddock, T., III. and Haines, Y. Y. 1975. A tax system for groundwater management. Water Resour. Res., 11: 7-14.

Morel-Seytoux, H. J., and Daly, C. J. 1975. A discrete kernel generator for stream-aquifer studies. Water Resour. Res., 11: 253-260.

Morel-Seytoux, H. J., Illangasekare, T. H. and Simpson, A. R. 1981. Modeling for management of a stream-aquifer system. Proc. of Water Forum '81. ASCE/San Francisco, CA/August 10-14, 1981, pp. 1342-1349.

Peralta, R. C. and Killian, P. J. 1985. Optimal regional potentiometric surface design: least-cost water supply/sustained groundwater yield. Trans. ASAE, 28: 1098-1107.

Peralta, R. C. and Kowalski, K. 1986. Optimizing the rapid evolution of target groundwater potentiometric surface attainment. Trans. ASAE, 29: 940-947.

Peralta, R. C., Dutram, P. W., Peralta, A. W. and Yazdanian, A. 1986. Saturated thickness for drought and litigation protection. Groundwater, 24: 357-364.

Peralta, R. C., Yazdanian, A., Killian, P. J. and Shulstad, R. N. 1985. Future Quaternary groundwater accessibility in the Grand Prairie - 1993. Bulletin No. 877, Agricultural Experiment Station, University of Arkansas, Fayetteville, AR, 37 p.

Smith, D. A., Farler, C., Hale, R. W., Stuart, C. A. Jr., Garner, C. R., Dorland, D. and Coats, R. E. Jr. 1983. Arkansas rice budgets, 1983. Cooperative Extension Service publication MP 206, University of Arkansas, Fayetteville, Arkansas.

Sniegocki, R. T. 1964. Hydrogeology of a part of the Grand Prairie region, Arkansas. Geological Survey Water-Supply Paper 1615-B, U. S. Dept. of the Interior, Washington D. C.

Stuart, C. A. Jr., Farler, C., Smith, D. A., Garner, C. R., Hale, R., Dorland, D. and Coats, R. E. Jr. 1983. 1983 Arkansas soybean budgets. Cooperative Extension Service publication MP 208, University of Arkansas, Fayetteville, Arkansas.

Verdin, K. L., Morel-Seytoux, H. J. and Illangasekare, H. J. 1981. User's manual for AQUISIM: FORTRAN IV programs for discrete

kernels generation and for simulation of an isolated aquifer behavior in two dimensions. HYDROWAR Program. Colorado State University, Ft. Collins, Colorado. 199 p.