



# PHOTONICS Research

## Kerr frequency combs in large-size, ultra-high- $Q$ toroid microcavities with low repetition rates [Invited]

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**By overcoming fabrication limitations, we have successfully fabricated silica toroid microcavities with both large diameter (of 1.88 mm) and ultra-high- $Q$  factor (of  $3.3 \times 10^8$ ) for the first time, to the best of our knowledge. By employing these resonators, we have further demonstrated low-threshold Kerr frequency combs on a silicon chip, which allow us to obtain a repetition rate as low as 36 GHz. Such a low repetition rate frequency comb can now be directly measured through a commercialized optical-electronic detector.** © 2017 Chinese Laser Press

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### 1. INTRODUCTION

Optical frequency combs act as frequency “rulers,” providing evenly spaced frequency lines. They have become useful tools in wide ranges of advanced scientific and applied fields [1–6]. The traditional optical frequency comb [1,2] is based on a large femtosecond mode-locked laser system that is difficult to integrate on a chip. Over the past decade, the rapid development of high- $Q$  microresonators has led to the rise of a new type of optical frequency comb based on Kerr-nonlinearity, known as the Kerr comb [7,8]. Formed via the process of cascaded four-wave mixing, Kerr combs have been implemented in different kinds of microresonators [7]. Since their invention, Kerr combs have attracted tremendous attention and have been generated in several material systems [8–16], including several on-chip platforms [17–20]. On-chip operation is essential for integration with other optical components. Because of their small mode volumes and high- $Q$  factors, these microresonator-based Kerr combs usually consume far less pump power, whereas typical threshold powers of several tens of microwatts have been reported for such optical frequency combs [8]. Because of their unique advantages over traditional optical frequency combs, as well as their potential for use in future applications, a number of significant studies have been conducted on such combs. These include comb noise characterization and dynamics [14], octave spanning [21,22], and time-domain characterizations, such as pulse shaping [19] and soliton formation inside the resonators [23,24]. In addition, Kerr combs have

been demonstrated under various wavelength regimes, including visible light [25–30] and the near-infrared [8–16] to mid-infrared [31,32] ranges, each of which provides further potential for unique applications. Numerical simulations of Kerr combs have also been developed in depth [33,34]. One of the most significant applications of Kerr combs is providing a link between microwave and optical frequencies, such as in optical clocks [3,35]. To realize practical implementation in such cases, the comb needs to be fully stabilized [36]. Moreover, both a low repetition rate that is detectable with commercial detectors and octave spanning are required in the comb. A recent research study has shown that, by using silica disk resonators, one can achieve the lowest possible repetition rate of 2.6 GHz and near-octave comb spanning with a 66 GHz repetition rate [20].

Although the Kerr optical frequency comb was first achieved in silica toroid microcavities [8], due to the fabrication difficulty, microtoroid cavities are conventionally limited to low quality factors at large sizes [36]. As such, both the repetition rate and the threshold powers of the generated Kerr combs are quite high, which is undesirable for practical applications and makes this platform gradually unpopular in the community. In this paper, we present a method of producing low-threshold Kerr combs in large toroid cavities with diameters up to 1.88 mm and  $Q$  factors greater than  $3 \times 10^8$ . So far, the largest toroid cavity obtained in previously reported studies had a diameter of just 760  $\mu\text{m}$ , and its corresponding optical  $Q$  factor was  $2 \times 10^7$  [36] due to the fabrication limitations on large-diameter toroids. By optimizing the fabrication process,

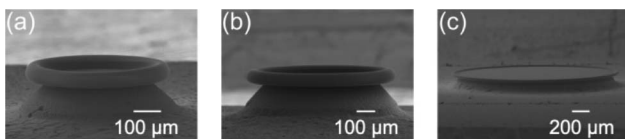
we achieved ultra-high- $Q$  millimeter scale toroid cavities with diameters and optical  $Q$  factors 2.5 and 16 times larger, respectively, than those of the previous results [36]. In addition, the cavities produced in our work possess higher optical  $Q$  factors than other ultra-high- $Q$  on-chip silica disk microcavities of the same size [37]. In the following, we describe three large toroid cavities of different sizes and demonstrate the generated optical frequency combs in each of them.

## 2. DEVICE FABRICATION AND MEASURED RESULTS

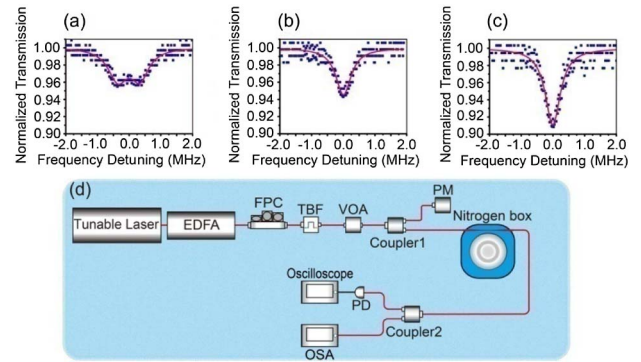
The process of fabricating our cavities is similar to the method used in Ref. [38] and can be briefly described as follows. First, we use photolithography and buffered HF wet etching to pattern a silica disk onto a silicon wafer. Then, we undercut the silica disk using XeF<sub>2</sub> dry etching to ensure that the silica disk is supported by a circular silicon pedestal. Finally, we use a CO<sub>2</sub> laser to reflow the silica disk.

Usually, to ensure the silica disk is molten during the reflow process, the undercut of the disk should be deep enough, since the silicon pillar acts as a heat sink to the absorbed laser power. However, due to the large stress (especially for a large diameter silica disk) of the thermal oxide, deep undercut of the silica disk will make the disk buckle, featuring a crown-like pattern [39]. This effect is more serious for the larger diameter silica disk with a thinner thickness, which limits the fabrication of a silica toroid cavity with a large diameter.

To overcome this problem, here, we employ a thick oxide film (12  $\mu\text{m}$  thickness) for the fabrication of large-diameter toroid cavities. Also, the thick thickness of the thermal oxide is helpful for the absorption of the CO<sub>2</sub> laser during the reflow process. During the experiment, the typical optical power of the CO<sub>2</sub> laser is 25–30 W. It is worth mentioning that, unlike repeating the reflow method of simply circulating the laser around the rim of the disk, as used in a previous study [36], when fabricating the toroid with a diameter of 514 or 800  $\mu\text{m}$ , we illuminate the CO<sub>2</sub> laser over the whole surface of the disk so that its rim can be melted more evenly. However, when fabricating a toroid with a diameter of 1.88 mm, due to both the limited power of the CO<sub>2</sub> laser and the limited cross-sectional size of the laser beam, we circulate the laser around the disk's rim. Figure 1 shows scanning electron microscopy (SEM) images of our large toroid resonators. The principle and minor diameters of the three fabricated toroid cavities are 514  $\mu\text{m}$ , 800  $\mu\text{m}$ , and 1.88 mm; and 81.4, 78.7, and 70.6  $\mu\text{m}$ , respectively. Accordingly, their calculated mode volumes for the fundamental TE-like cavity mode are  $3.2 \times 10^{-14}$ ,  $5.5 \times 10^{-14}$ , and  $22.8 \times 10^{-14}$  m<sup>3</sup>, respectively [40].



**Fig. 1.** SEM images of toroid cavities: (a) cavity with principle diameter of 514  $\mu\text{m}$ , (b) cavity with principle diameter of 800  $\mu\text{m}$ , and (c) cavity with principle diameter of 1.88 mm.



**Fig. 2.** (a) Transmission spectrum of toroid cavity with a diameter of 514  $\mu\text{m}$  and the corresponding intrinsic  $Q$  factor of  $2.7 \times 10^8$ ; (b) transmission spectrum of a cavity with diameter of 800  $\mu\text{m}$  and the corresponding intrinsic  $Q$  factor of  $3.0 \times 10^8$ ; (c) transmission spectrum of cavity with a diameter of 1.88 mm and the corresponding intrinsic  $Q$  factor of  $3.3 \times 10^8$ ; (d) experimental setup. EDFA, erbium doped fiber amplifier; FPC, fiber polarization controller; TBF, tunable bandpass filter; VOA, variable optical attenuator; PM, powermeter; PD photodiode; and OSA, optical spectrum analyzer.

We test our sample's  $Q$  factor, threshold power, and comb formation using the setup shown in Fig. 2(d). The  $Q$  factor of our sample is determined by first transmitting the scanned laser (with a linewidth of  $\sim 200$  kHz) into the cavity through a tapered fiber and then measuring the full width at half-maximum of the transmitted Lorentz peak using an oscilloscope. The intrinsic  $Q$  factors are measured to be  $2.7 \times 10^8$ ,  $3.0 \times 10^8$ , and  $3.3 \times 10^8$  for cavities with diameters of 514  $\mu\text{m}$ , 800  $\mu\text{m}$ , and 1.88 mm, respectively, as shown in Figs. 2(a)–2(c). The achieved optical  $Q$  factors and the diameters of the toroid cavities are 16.5 and 2.5 times larger, respectively, than the ones reported in the previous work [36]. Also, these measured optical  $Q$  factors are  $\sim 3.0$ , 1.7, and 1.2 times larger than the wet-etched silica disk cavities with similar diameters [20,37]. To the best of our knowledge, our cavities have the highest  $Q$  factors among all the reported on-chip silica microresonators of the same sizes.

## 3. DISPERSION SIMULATION

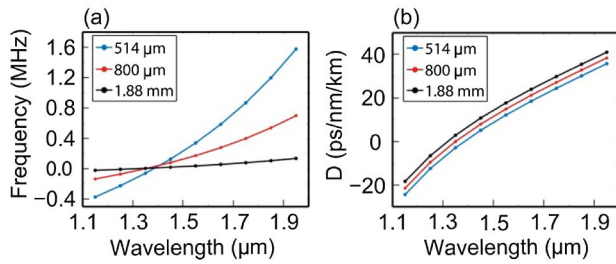
Before generating optical frequency combs, we calculate the dispersion of our toroid cavities through the finite-element-simulation method with COMSOL 4.3 software [40]. The dispersion can be determined via the detuning of the cavity's two adjacent free spectral ranges (FSRs), which we define as

$$\Delta\text{FSR} = (\nu_{m+1} - \nu_m) - (\nu_m - \nu_{m-1}) = \nu_{m+1} + \nu_{m-1} - 2\nu_m. \quad (1)$$

We can also describe the dispersion as [8]

$$D \approx \frac{\Delta\text{FSR} \times 4\pi^2 n^3 R^2}{c^2 \lambda^2}, \quad (2)$$

where  $R$  stands for the radius of the cavity. In Figs. 3(a) and 3(b), we show the calculated dispersions of these three cavities. It is clear that all three cavities possess anomalous dispersions ( $D > 0$ ) under the 1550 nm wavelength regime, which is advantageous for the generation of Kerr combs.



**Fig. 3.** (a) Calculated dispersions represented as  $\Delta\text{FSR}$  and (b) calculated dispersions represented as  $D$  (in ps/nm/km).

#### 4. THRESHOLD POWER MEASUREMENT

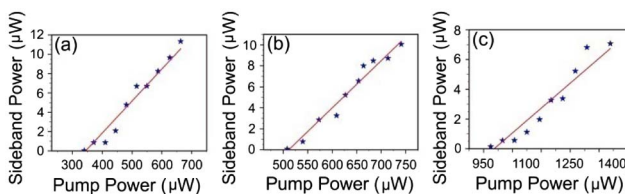
Then, we measure the thresholds of optical parametric oscillation of our toroid cavities. As shown in Fig. 4, the measured threshold powers for toroid cavities with diameters of 514 μm, 800 μm, and 1.88 mm are 330, 508, and 980 μW, respectively, under their own coupling conditions. These values are lower than the threshold powers of both on-chip silica disks and toroids at similar sizes due to the increased  $Q$  factors [20,36]. According to previous studies [14], the formula for the threshold power of optical parametric oscillation in a whispering gallery mode cavity can be written as

$$P_{\text{th}} = \frac{V_0 n_0^2 \omega}{8n_2 c \eta Q^2}, \quad (3)$$

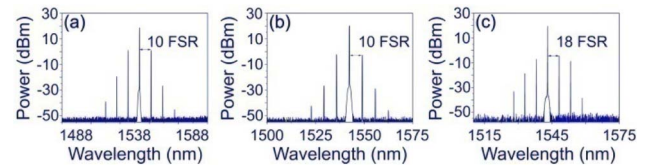
where  $V_0$  denotes the effective mode volume,  $n_0$  represents the linear refractive index of silica,  $\omega$  is the pump laser's angular frequency,  $n_2$  is the nonlinear refractive index of silica,  $\eta$  is the coupling rate,  $c$  is the speed of light in vacuum, and  $Q$  represents the total  $Q$  factor of the pump mode. By substituting parameters into Eq. (3), the calculated threshold values match well with the experimental data.

#### 5. KERR COMB GENERATION

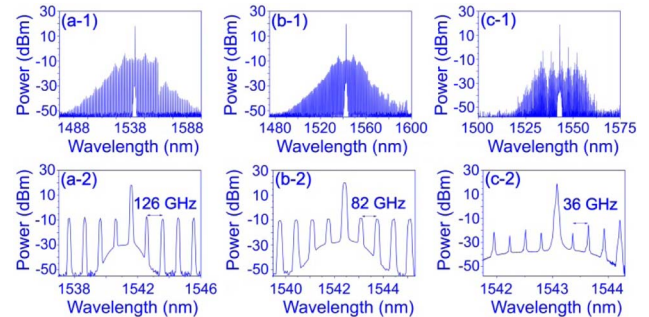
Next, we illustrate the dynamic comb-generation process as we tune the wavelength from just inside the resonance mode toward the resonance point. During this process we use the thermal lock method [41] to ensure that the wavelength does not shift away, and to achieve mechanical stability, we attach a tapered fiber to the cavity so that the coupling rate between the cavity and the fiber remains constant during the process. Naturally, all three



**Fig. 4.** (a) Measured threshold power of toroid with diameter of 514 μm, loaded  $Q$  factor of  $8.5 \times 10^7$ , and corresponding coupled pump power of 330 μW. (b) Measured threshold power of toroid with diameter of 800 μm, loaded  $Q$  factor of  $2.4 \times 10^8$ , and corresponding coupled pump power of 508 μW. (c) Measured threshold power of toroid with diameter of 1.88 mm, loaded  $Q$  factor of  $1.7 \times 10^8$ , and corresponding coupled pump power of 980 μW.



**Fig. 5.** First comb lines excited for (a) 514 μm toroid, first comb line excited 10 FSR away from the central mode; (b) 800 μm toroid, first comb line excited 10 FSR away from the central mode; and (c) 1.88 mm toroid, first comb line excited 18 FSR away from the central mode.



**Fig. 6.** Spectra of optical combs: (a-1) optical frequency comb in toroid with diameter of 514 μm, and corresponding repetition rate of 126 GHz; (a-2) closeup of nine comb lines around the pump mode; (b-1) optical frequency comb in toroid with diameter of 800 μm, and corresponding repetition rate of 82 GHz; (b-2) closeup of nine comb lines around the pump mode; (c-1) optical frequency comb in toroid with diameter of 1.88 mm, and corresponding repetition rate of 36 GHz; (c-2) closeup of nine comb lines around the pump mode.

cavities operate in the over-coupled regime. As Fig. 5 illustrates, in each of these cases, the first comb line is not excited in the mode adjacent to the pump mode and that corresponds to a Turing pattern comb [34]. The first line to become excited can be simulated by the method presented in Refs. [33,34].

Figure 6 shows the measured spectra of the generated frequency comb using an optical spectrum analyzer with a resolution of 0.02 nm. The numbers of comb lines are 118, 149, and 146 with coupled pump powers of 15.4, 18.7, and 20.1 mW, respectively, for resonators with diameters of 514 μm, 800 μm, and 1.88 mm. In the cavities with diameters of 800 μm and 1.88 mm, the combs have repetition rates of 82 and 36 GHz, respectively, which can be detected directly with a commercial detector and is very useful when locking methods are used to attempt to stabilize the comb [36]. To our knowledge, in the structure of the silica toroid microcavities, the 36 GHz repetition rate is the lowest repetition rate that has ever been achieved.

#### 6. CONCLUSIONS

We demonstrated the generation of optical frequency combs in large toroid microresonators of different sizes whose  $Q$  factors exceeded  $3.0 \times 10^8$  and whose diameters reached as large as

1.88 mm. The threshold power of our biggest device was lower than 1 mW with a repetition rate of 36 GHz, which, to the best of our knowledge, is the smallest repetition rate ever achieved in a silica toroid cavity. A commercial detector was able to detect this value directly, which better facilitates its practical applications. In future studies, we will analyze the noise of the combs in our cavities and attempt to stabilize the combs by locking the pump lasers, both in amplitude and frequency, and generate the solitons. Beyond the generation of the frequency comb, such large toroid cavities with ultra-high- $Q$  factor can also be very useful in demonstrating narrow-linewidth Brillouin lasers, on-chip frequency stabilization, and gyroscopes.

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